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Quality of Schooling: Child Quantity-Quality Tradeoff, Technological Progress and Economic Growth

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Abstract

An overlapping generations version of an R&D-based growth model 'a la Diamond (1965) and Jones (1995) is built to examine how improvement in quality of schooling impact technical progress and long- run economic growth of an economy by influencing fertility and education decisions at household level. The results indicate that improvement in schooling quality triggers a child quantity-quality trade-off at household level when quality of schooling exceeds an endogenously determined threshold. At the household level, parents invest more in education of children and have lesser number of children in response to improvement in quality of schooling. This micro-level tradeoff has two opposing effects on aggregate human capital accumulation at macro level. Higher investment in education of a child stimulates the accumulation of human capital which fosters technical progress but the simultaneous decline in fertility rate reduces the total factor productivity growth and economic growth by contracting the pool of available researchers. The first effect prevails over latter only when quality of schooling is higher than the threshold.

Keywords— fertility, quality of schooling, economic growth, demographic transition

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1 Introduction

Human capital as a potential driver of technical change has, consequently, emerged as an important determinant of economic growth in the endogenous growth theory literature. The policy emphasis on schooling in the development strategies of most countries mirrors the emphasis of research on the role of human capital in determining growth and development pattern of economies. Developing countries have focussed on improving the access to education so that their stock of human capital can be built up which, in turn, can be fruitfully employed to speed up the process of technological progress and diffusion and, therefore, spur economic growth. As per the estimates of Barro and Lee (2013), the share of population without any formal schooling in developing countries has declined from 54.6 percent in 1960 to approximately 17.4 percent in 2010. However, merely expanding access to education does not ensure that children actually learn in schools. The learning outcomes in schools closely hinge upon the quality of schooling, which has been given inadequate attention in the development policy paradigms of most developing countries until now. But recently, development policy paradigms of most countries are gradually shifting towards improving learning quality in schools than merely expanding access to education. This policy paradigm shift in education policy is also reflected in the post-2015 development agenda. Imparting quality education features as the fourth Sustainable Development Goal set by the United Nations. This shift is motivated by two factors.

First, there is growing evidence that quality of schooling matters more for economic growth. Hanushek and Kimko (2000) and Hanushek and Woessmann (2012) provide an extensive discussion of how scores from cognitive skill tests can be used to measure the quality of human capital and its effects on economic growth. They use data from six voluntary international tests of mathematics and science to build a measure of quality of education. They find that the estimate of human capital quality has a significant positive impact on growth. Several studies have since found very similar results (Bosworth & Collins, 2003; Ciccone & Papaioannou, 2009; Islam, Ang, & Madsen, 2014a). Second, poor quality of schooling remains a dismal reality in developing countries. UNESCO (2014) reports that 250 million children are functionally illiterate and innumerate despite 50 percent of them having spent at least four years in school. According to the Annual Status of Education Report (ASER) 2017 survey titled "Beyond Basics", based on an assessment of 30,000 children in

28 districts of 24 states in India, only 43 percent of 14-18-year-olds could do simple division after eight years of schooling. Less than half of the children surveyed could not add weights in kilograms and more than 40 percent could not tell hours and minutes from a clock. 36 percent of the children could not answer the capital of India correctly. This scenario is staggering and reveals a gloomy picture about the learning outcomes of children in Indian schools. Similarly, Glewwe, Ilias, and Kremer (2010) report that teachers from rural schools in Kenya were absent 20 percent of the time; while, in Zambia and Pakistan, teachers were absent, respectively, 18 percent and 10 percent of the time (J. Das, Dercon, Habyarimana, & Krishnan, 2004; Reimers, 1993). This implies that poor quality of schooling significantly distorts the learning outcomes in schools, which in turn, has far-reaching implications on growth prospects of developing countries.

In this paper, we build an overlapping generations version of an R&D-based growth model á la Diamond (1965) and Jones (1995) to analyze how improvement in quality of schooling and the associated changes in fertility and education decisions at the micro level influence the long-run economic growth of an economy. We characterize two types of economies. The first type is an innovation economy where technological improvements occur by innovating upon the local technology frontier. The second type is an imitation economy where technological progress occurs by imitating existing foreign technologies. We find that the quality of schooling triggers a child quantity-quality trade-off at the micro level when quality of schooling surpasses an endogenously determined threshold under both the technology regimes. When quality of schooling surpasses the threshold, parents invest in the education of their children and bear lesser number of children. However, parents focus on maximizing fertility and do not educate their children when quality of schooling is less than the threshold. This micro-level trade-off generates two types of effects on economic growth at the macro level - a growth-stimulating effect and a growth-impeding effect. Higher investment in the education of a child stimulates the accumulation of human capital, which fosters technical progress but the simultaneous decline in fertility rate reduces the total factor productivity growth and economic growth by contracting the pool of available researchers. Our results show that the former effect dominates over latter only when the quality of schooling is higher than the threshold, and the economy is on a self-sustaining growth path. Alternatively, when the quality of schooling is less than the threshold, parents do not educate their children and focus, instead on maximizing fertility. In such a scenario, economic growth is solely driven by quantity of human capital. Higher fertility rate leads to higher population growth, which propels economic growth rate under both innovation and imitation regimes.

Our theoretical result that improvement in quality of schooling leads to higher investment in education when quality of schooling surpasses an endogenously determined threshold is consistent with recent theoretical and empirical findings. In a theoretical context, Castelló-Climent and Hidalgo-Cabrillana (2012) study the effects of exogenously determined school quality on student choices of education and its consequent impact on economic growth. They find that high-quality education increases the returns to schooling, and hence, the incentives to accumulate human capital. They also emprically validate their theoretical result by carrying out a cross-country analysis which reveals that quality of education has a positive effect on enrollment rates in secondary schooling only when quality of schooling is sufficiently high. Similarly, Hanushek, Lavy, and Hitomi (2008) find that lower quality of schooling leads to higher dropout rates in case of Egyptian primary schools.

Our work is closely related to two broad strands of literature. First, there is theoretical literature that analyzes the linkages between quality of schooling and economic growth. Many existing studies (M. Das & Guha, 2012; Gilpin & Kaganovich, 2012; Tamura, 2001) on quality of schooling and economic growth focus on explaining how determinants of quality of schooling such as teacher-student ratio and teacher quality together impact the learning process, and the consequent human capital formation and, therefore, economic growth. M. Das and Guha (2012) consider an economy with heterogenous agents. In their framework, two teacher specific inputs (teacher quality and teacher quantity) and two student-specific inputs (ability and effort) enter the production function of human capital. The human capital accumulation and economic growth depends upon this complex interaction between these two types of schooling inputs. Similar to M. Das and Guha (2012), Gilpin and Kaganovich (2012) analyze a similar quantity-quality trade-off of teachers in a two-tiered schooling system: basic and advanced (college level). Their results show that the hiring costs of teachers increase over time leading to a shift of the optimal trade-off betwen quality and quantity in favor of the latter in the process of endogenous growth. Castelló-Climent and Hidalgo-Cabrillana (2012) develop a theory of human capital investment to study the effects of exogenously determined schooling quality on student choices of education, and to understand its effect on economic growth. High-quality education increases the returns to schooling, and hence the incentives to accumulate human capital. This is caused by two different channels: higher quality incentivizes people to acquire education (extensive margin), and once individuals decide to participate in higher education, higher-quality increases the investment made per individual (intensive margin). These two channels, in turn, lead to economic growth.

However, most of these studies assume exogenously determined population growth and do not consider technical progress in their models. Consequently, these studies are unable to analyze the impact of schooling quality and the resulting demographic change on R&D activities, which are a major determinant of technological development in the present world. We improve upon these papers by endogenizing both - population growth and technical change. Specifically, our work focuses on interactions between quality of schooling and demographic change, which influence total factor productivity growth and, therefore, growth prospects of an economy.

Second, our research relates to the literature linking R&D based growth with endogenous fertility and education decisions (Hashimoto & Tabata, 2016; Strulik, 2005; Strulik, Prettner, & Prskawetz, 2013). Strulik (2005) introduce human capital accumulation in a R&D-based growth model where R&D activity is driven by expansion in variety and quality of intermediate inputs. He finds that economic growth depends positively on the rate of human capital accumulation and positively or negatively on population growth depending on the degree of altruism towards future generations. Economic growth and population growth are negatively correlated if households maximize utility derived from their own per capita consumption. Alternatively, economic growth and population growth are positively correlated if households maximize utility derived from the consumption of their dynasty. Hashimoto and Tabata (2016) examine how increase in the old-age survival rate influences fertility and education decisions at the micro level and its consequent impact on economic growth at the macro level. They show that an increase in life expectancy encourages young individuals to invest more in their education and bear fewer children at the household level. Further, they show that in economies in which life expectancy is sufficiently low, this micro-level trade-off yields a higher rate of human capital accumulation and, therefore, higher rate of technical progress and economic growth at the macro level. However, in economies in which life expectancy is sufficiently high, this micro-level trade-off leads to greater decline in population growth rate which impedes the increase in the supply of researchers and, thereby, reduces the rate of technical progress and economic growth of the economy.

In particular, this work is closely related to Strulik et al. (2013). Strulik et al. (2013) analyze child quantity-quality trade-off by integrating R&D based innovations into a unified growth framework. They explain why high levels of total factor productivity and economic growth in modern economies are associated with low or negative population growth by considering a child quantity-quality trade-off at the household level. In their theoretical model, decisions related to fertility and education are endogenously determined by households. A substitution of child quantity, n, by child quality (i.e. expenditure on education), e, that keeps total child expenditure, $e.n$, constant sets free parental time, which can be used to earn extra income. The additional income is partly spent on education, such that the overall child expenditure rises more proportionately than child quantity falls. On the macro side of the economy, this trade-off means that the magnitude by which human capital per person, h rises is larger than the magnitude by which number of persons, L, falls. The net impact of this micro level trade-off is that the total available human capital $h.L$ increases at the macro level. Given that human capital is the driving force for R&D, this entails a higher R&D output and higher R&D-based growth.

Although our modelling framework is similar to Strulik et al. (2013), we go beyond Strulik et al. (2013) in atleast three respects. First, the focus of our research is not on formulating a unified growth theory which explains the entire transition of an economy from Malthusian stagnation to modern growth. Instead, the purpose of this work is to build a growth model that explains the inter-linkages between quality of schooling, demographic change and technological improvements in a modern economy. Therefore, this thesis focusses on characterizing two types of economies with low and high quality of schooling and examines the corresponding drivers of economic growth in these two types of economies. To the best of our knowledge, this issue is yet to be explicitly discussed in the literature. Second, our study examines the impact of a demographic transition triggered by improvement in quality of schooling. Strulik et al. (2013) focus on impact of a demographic transition induced by technological progress. Third, we extend Strulik et al. (2013) by considering two distinct channels of technological improvement - innovation and imitation. Under the innovation regime, technological improvements occur by innovating on local technology frontier whereas under imitation regime, technological progress occurs by imitating existing foreign technologies. In this respect as well, this work is an improvement over existing research in this area.

This paper is organized as follows. Section 2 discusses the basic structure of the model. Section 3 contains the key analytical results for a decentralized economy, which provide the key propositions of this study. Section 4 concludes.

2 The Model and Equilibrium Solutions

2.1 The Economic Environment

We consider a model economy populated by overlapping generations of people who live for two periods: adulthood and old age. Time is discrete and goes from 0 to ∞. During childhood, which is not modeled explicitly, individuals are reared and educated by their parents. All the decisions are made at the beginning of adulthood. Adults are identical in all aspects. They inelastically supply their skills in the labor market. Adults care about consumption of a homogeneous final good, number and human capital level of their children. During old age, individuals consume their savings plus interest earned on these. Abstracting from gender differences, each household has a single parent. For avoiding the indivisiblity problem, we assume that children are in continuous number. All individuals survive up to adulthood. The education of current period's children determines human capital endowment of next period's adult generation. Akin to Castelló-Climent and Hidalgo-Cabrillana (2012), human capital accumulation function depends on an exogenously given quality of education system, parental investment in education and human capital of parent. Parental investment in education is a fraction of income spent on education of each child.

The production structure of the economy closely follows Romer (1990) and Jones (1995). The economy consists of three sectors: final goods sector, intermediate goods sector and R&D sector. R&D sector employs human capital to produce blueprints of intermediate goods. Intermediate goods are produced by monopolistic firms using physical capital and intermediate good-specific blueprint. Final goods sector produces the good competitively using land and variety of intermediate goods as inputs.

2.2 Individuals

Individuals derive utility from $c_{1,t}$, their own consumption (of the final good) during adulthood; $c_{2,t+1}$, their own consumption during old age; n_t, number of children and h_{t+1}, human capital of children. Parents' motivation to invest in human capital of children by spending on children's education is driven by a "warm glow" of giving (Andreoni, 1989) or preference for having "higher-quality" children (Becker, 1960). The lifetime expected utility of individuals in generation t is given by:

$$u_t = \log c_{1,t} + \beta_1 \log c_{2,t+1} + \beta_2 \log(h_{t+1} n_t), \tag{1}$$

where positive weights β_1 and β_2 measure the importance of future consumption and child quantity and quality relative to current consumption in the utility function. Alternatively, following De la Croix and Doepke (2004), β_2 can be interpreted as an altruism factor.

An adult's human capital is denoted by h_t and the wage per unit of human capital is w_t. Young adults spend their income on current consumption, savings for old-age consumption and child's education expenditure. Rearing a child necessarily takes fraction $\tau \in (0,1)$ of an adult's time, which is given exogenously. Accordingly, the budget constraints for the young and old adults are given by:

$$w_t h_t (1 - \tau n_t) = c_{1,t} + s_t + e_t (w_t h_t) n_t; \tag{2}$$

$$c_{2,t+1} = (1 + r_{t+1}) s_t, \tag{3}$$

where e_t is the fraction of income per child spent on education, s_t is savings and r_{t+1} is interest rate. Non-negativity constraints apply to all the variables.

The human capital of children, h_{t+1}, depends on human capital of parents, h_t, parental investment in education per child, e_t, and quality of education system, θ, which is exogenously given.

$$h_{t+1} = (\mu + \theta e_t)^{\epsilon} h_t, \qquad \epsilon < 1. \tag{4}$$

The parameters satisfy $\mu \geq 1$ and $\epsilon \in (0,1)$. ϵ measures the returns to education. μ is the intergenerational human capital spillovers that are basically skills learnt by children by observing and imitating parents. The parametric restriction of $\mu \geq 1$ ensures that the growth rate of per capita human capital does not become negative when parents do not invest in education. It ensures that children will acquire knowledge and skills atleast equivalent to their parents when parents do not educate their children.The assumption that quality of schooling is an argument in human capital accumulation function is consistent with a number of studies. Hanushek et al. (2008) find that lower-quality schools lead to higher dropout rates in case of Egyptian primary schools. Similarly, Hanushek and Woessmann (2008) find that cognitive skills, a proxy for educational quality, is positively related to individual earnings. In theoretical terms, Castelló-Climent and Hidalgo-Cabrillana (2012) have shown in their theory of human capital investment that high-quality education increases the returns to schooling and incentivizes human capital accumulation via two channels- extensive and intensive margins. Higher quality makes education accessible to more people (extensive margin), and once individuals decide to participate in higher education, higher quality increases the investment made per individual (intensive margin). Parental human capital, h_t, as an input in human capital accumulation technology represents intergenerational transfers of human capital, which is a common assumption in the literature (De la Croix & Doepke, 2004; Kalemli-Ozcan, 2002, 2003; Tamura, 2001).

Individuals maximize utility in eq. (1) with respect to the constraints, eqs. (2) to (4) using control variables $c_{1,t}$, s_t, n_t and e_t. The solution to individuals' decision problem can either be interior, or at a corner where the individuals choose zero education. The first-order conditions yield the following solution, as in eqs. (5) to (8), for consumption and savings irrespective of whether education is in the interior or at the corner:[1]

$$c_{1,t} = \frac{w_t h_t}{1 + \beta_1 + \beta_2}; \tag{5}$$

$$s_t = \frac{\beta_1 w_t h_t}{1 + \beta_1 + \beta_2}. \tag{6}$$

For child quantity and quality, there exists a threshold level of quality of schooling. If quality of schooling falls below the threshold, adults do not spent on child quality and maximize child quantity. This constitutes the corner solution. In particular, following results are derived from the

[1]Detailed mathematical derivations are provided in Appendix A.

first-order conditions:

$$e_t = \begin{cases} 0, & \text{if } \theta \leq \frac{\mu}{\tau\epsilon}; \\ \dfrac{\tau\theta\epsilon - \mu}{\theta(1-\epsilon)}, & \text{otherwise,} \end{cases} \tag{7}$$

$$n_t = \begin{cases} \dfrac{\beta_2\epsilon\theta}{(1+\beta_1+\beta_2)\mu}, & \text{if } \theta \leq \frac{\mu}{\tau\epsilon}; \\ \dfrac{\beta_2\theta(1-\epsilon)}{(1+\beta_1+\beta_2)(\tau\theta-\mu)}, & \text{otherwise.} \end{cases} \tag{8}$$

Inserting eq. (7) in eq. (4), we get an equation of motion for human capital as:

$$h_{t+1} = \begin{cases} \mu^\epsilon h_t, & \text{if } \theta \leq \frac{\mu}{\tau\epsilon}; \\ \left[\dfrac{\epsilon(\tau\theta-\mu)}{(1-\epsilon)}\right]^\epsilon h_t, & \text{otherwise.} \end{cases} \tag{9}$$

Below the threshold, quality of schooling is not an argument in human capital production function. Without education expenditure, human capital of next generation consists of basic skills only. From eqs. (5) to (8), irrespective of whether quality of schooling exceeds threshold or not, savings and consumption are increasing in $w_t h_t$ and there is no direct effect of income on fertility because a positive income effect of an increase in wages on fertility is balanced by a negative substitution effect. The quality of schooling has a direct bearing on child quantity and quality. The following lemma shows how quality of schooling influences fertility behavior.

Lemma 1 *When quality of schooling is high enough to surpass the threshold, a marginal improvement in the quality of schooling triggers a child quantity-quality trade-off such that adults bear lesser number of children and invest more in education per child in response to improvement in quality of schooling. However, when quality of schooling is lower than the threshold, then it has no effect on child quality as adults do not invest in child's education and focus instead on maximizing child quantity.*

Proof. By investigating the corner solution in eqs. (7) and (8), it can be immediately seen that quality of schooling entails no child quantity-quality trade-off if quality of schooling falls below the threshold. Adults do not spend on education and maximize fertility. To see the effect when quality of schooling is above the threshold, we take the derivatives of the interior solution of e_t and n_t with respect to θ in eqs. (7) and (8). That is,

$$\frac{\partial n_t}{\partial \theta} = \frac{-\mu\beta_2(1-\epsilon)}{(1+\beta_1+\beta_2)(\tau\theta-\mu)^2} < 0;$$

$$\frac{\partial e_t}{\partial \theta} = \frac{\mu}{(1-\epsilon)\theta^2} > 0.$$

When quality of schooling is less than the threshold, the derivatives of the corner solution of e_t and n_t with respect to θ in eqs. (7) and (8) yield:

$$\frac{\partial n_t}{\partial \theta} = \frac{\beta_2 \epsilon}{(1+\beta_1+\beta_2)\mu} > 0;$$

$$\frac{\partial e_t}{\partial \theta} = 0.$$

■

Thus, it can be seen that fertility changes are directly triggered by quality of schooling. Any improvement in quality of schooling over and above the threshold makes learning in schools more effective and, therefore, increases marginal returns to investment in human capital. Consequently, a parent reduces fertility and spends more on education per child. Thus, quality of schooling can be perceived as another plausible mechanism for triggering child quantity-quality trade-off besides other commonly proposed mechanisms such as declining child mortality (Soares, 2005), rise in life expectancy of parents (Boucekkine, Croix, & Licandro, 2003; Boucekkine, De la Croix, & Licandro, 2002; Hashimoto & Tabata, 2016; Kalemli-Ozcan, 2002, 2003), technical progress (Galor & Weil, 2000) and decline in gender wage gap (Galor & Weil, 1996). These theoretical results are in line with recent empirical findings. For example, Hanushek et al. (2008) find that lower quality of schooling leads to higher dropout rates in Egyptian primary schools. A cross-country analysis by Castelló-Climent and Hidalgo-Cabrillana (2012) reveals that quality of education has a positive effect on enrollment rates in secondary schooling only when quality of schooling is sufficiently high.

Lemma 2 *An increase in returns to education, ϵ, leads to a child quantity-quality trade-off wherein parents educate their children and bear lesser number of children when quality of schooling surpasses the threshold. However, when quality of schooling is less than the threshold, returns to education has no effect on education of children and parents maximize child fertility.*

Proof. Taking the derivatives of the interior solution of e_t and n_t with respect to ϵ in eqs. (7) and (8), one gets that:

$$\frac{\partial n_t}{\partial \epsilon} = \frac{-\beta_2 \theta}{(1+\beta_1+\beta_2)(\tau\theta-\mu)} < 0;$$

$$\frac{\partial e_t}{\partial \epsilon} = \frac{\tau\theta-\mu}{\theta(1-\epsilon)^2} > 0.$$

When quality of schooling is less than the threshold, the derivatives of the corner solution of e_t and n_t with respect to ϵ in eqs. (7) and (8) yield:

$$\frac{\partial n_t}{\partial \epsilon} = \frac{\beta_2 \theta}{(1+\beta_1+\beta_2)\mu} > 0;$$

$$\frac{\partial e_t}{\partial \epsilon} = 0.$$

■

This implies that returns to education is yet another factor that can trigger a child quantity-quality trade-off. High returns to education implies education makes human capital more productive. Therefore, parents invest in education of their children and decide to have lesser number of children. However, when quality of schooling is less than the threshold, then parents decide not to make any investment in the education of children and, therefore, returns to schooling has no effect on child quality and child quantity is maximized. Both Lemmas 1 and 2 will be used later in our analysis.

2.3 Final Goods Sector

The final homogenous good, Y_t is produced and sold in a competitive market. For any firm, the production structure at time t is defined as:

$$Y_t = l_t^{1-\alpha} \sum_{i=1}^{A_t} x_{it}^{\alpha}, \qquad 0 < \alpha < 1. \tag{10}$$

The production of final good uses land and a variety of intermediate inputs. For simplicity, the total supply of land, l_t, is kept fixed and has been normalized to 1. $x_{i,t}$ is the quantity of ith intermediate input that is used in the final goods production and A_t is the number of available varieties of intermediate inputs or the level of technological knowledge that grows through R&D. The parameter α is the capital share in final goods production. This production specification exhibits diminishing marginal productivity in each input, l_t and $x_{i,t}$, and constant returns to scale in all inputs together. The existence of additive separability across $x_{i,t}$ implies that the marginal product of intermediate input i is independent of the quantity employed of intermediate input $i+1$. Thus, a new type of intermediate good is neither a direct substitute for nor a direct complement of the types that already exist. Therefore, discoveries of new types of intermediates do not make any existing types obsolete. The price of final good P_Y has been normalized to 1. In each period t, the final good producers solve the following profit maximization problem with respect to their choice of range of intermediate inputs:

$$Max_{x_{it},l_t} \quad \pi_t(Y) = l_t^{1-\alpha} \sum_{i=1}^{A_t} x_{it}^{\alpha} - \sum_{i=1}^{A_t} p_{it} x_{it} - d_t l_t, \tag{11}$$

where p_{it} is the unit monopoly price of ith intermediate input and d_t is the rate of return on land. The first-order conditions imply that:

$$p_{it} = \alpha l_t^{1-\alpha} x_{it}^{\alpha-1}. \tag{12}$$

Eq. (12) yields the demand for each intermediate input as

$$x_{it} = \left[\frac{\alpha}{p_{it}}\right]^{\frac{1}{1-\alpha}} l_t; \tag{13}$$

$$d_t = (1-\alpha) l_t^{1-\alpha-1} \sum_{i=1}^{A_t} x_{it}^{\alpha} = \frac{(1-\alpha)Y_t}{l_t}. \tag{14}$$

where the last expression is deduced after substituting for Y_t from eq. (10). We have not explicitly modeled the government sector. However, to keep the exposition simple, it has been assumed that the total return on land is paid by the households to the government.

An analysis of the intermediate goods sector ensues.

2.4 Intermediate Goods Sector

Each intermediate good i is produced by monopolist producer who holds the blueprint to produce x_{it} quantity at time t. Each intermediate good uses only capital in a one-to-one production technology, or $x_{it} = K_{it}$. Thus, the amount of intermediate inputs produced of all types equals the aggregate capital stock of the economy.

$$\sum_{i=1}^{A_t} x_{it} = K_t. \tag{15}$$

Each ith intermediate good producer maximizes profits with respect to his/her choice of capital. That is,

$$Max_{x_{it}} \quad \pi_t(i) = p_{it} x_{it} - r_t K_{it} = \alpha l_t^{1-\alpha} x_{it}^{\alpha} - r_t x_{it}, \tag{16}$$

where the expression in the r.h.s derives from substituting the solution to p_{it} from eq. (12) and $x_{it} = K_{it}$. r_t is the price per unit capital. The first-order condition leads to

$$\alpha^2 l_t^{1-\alpha} x_{it}^{\alpha-1} = r_t. \tag{17}$$

Using eq. (12), we get the solution to equilibrium price as $p_{it} = p_t = \frac{r_t}{\alpha}$. This is the monopoly price charged as a markup over marginal cost. Note that being independent of i, this price is constant across all intermediate goods. From eq. (13), this implies that quantity produced of each i is the same, that is, $x_{it} = x_t = \left[\frac{\alpha^2}{r_t}\right]^{\frac{1}{1-\alpha}} l_t$. In equilibrium, the net profit of the ith monopolist is given by:

$$\pi_t = p_t x_t - r_t x_t \equiv \left[\frac{r_t}{\alpha} - r_t\right] x_t \equiv \left[\frac{1-\alpha}{\alpha}\right] r_t x_t; \tag{18}$$

$$= \alpha(1-\alpha) l_t^{1-\alpha} x_t^{\alpha}, \tag{19}$$

where last expression has been derived using eq. (17) and $x_{it} = x_t$ at the equilibrium. Since, in equilibrium, intermediate inputs are sold at the same price and demanded in equal quantities, aggregate physical capital is given by $K_t = A_t x_t$. Inserting this information into the production function of the final good, eq. (10) simplifies to

$$Y_t = l_t^{1-\alpha} A_t^{1-\alpha} K_t^{\alpha}. \tag{20}$$

Accordingly, equilibrium profits of the ith monopolist in eq. (3.19) can be expressed as:

$$\pi_t = \alpha(1-\alpha)\frac{Y_t}{A_t}. \tag{21}$$

This follows after substituting for x_t from $K_t = A_t x_t$ and using eq. (20). Further, the price per unit of capital can be expressed as:

$$r_t = \alpha^2 l_t^{1-\alpha} \left[\frac{A_t}{K_t}\right]^{1-\alpha}. \tag{22}$$

Further using eq. (20), rental rate of capital can be simplified to:

$$r_t = \alpha^2 \left[\frac{Y_t}{K_t}\right]. \tag{23}$$

The R&D sector is now discussed.

2.5 R&D Sector

Under the assumption of free entry into the R&D sector, firms employ human capital to develop new blueprints which are sold at price, p_t^A, common to all blueprints due to competition in the market for blueprints. We consider two types of regimes that can drive R&D activities. The R&D sector produces blueprint of an intermediate variety either by imitating from the world technology frontier or by innovating upon the local technology level. Following Papageorgiou and Perez-Sebastian (2006) and Guilló, Papageorgiou, and Perez-Sebastian (2011), the production function of technology for a firm is postulated as:

$$A_{t+1} - A_t = \delta_t H_t, \tag{24}$$

where $A_{t+1} - A_t$ are new blueprints. Productivity of R&D activity, δ_t, is constant at the firm level but at the aggregate level, it is defined as:

$$\text{Innovation regime} : \delta_t = \bar{\delta} H_t^{\lambda-1} A_t^{\phi}; \tag{25}$$

$$\text{Imitation regime} : \delta_t = \bar{\delta} H_t^{\lambda-1} A_t^{\phi} \left[\frac{\bar{A}_t}{A_t} \right]. \tag{26}$$

R&D productivity depends positively on the number of already existing ideas, A_t, and human capital employed in R&D sector, H_t. The parameter $\bar{\delta}$ denotes general productivity in R&D. $0 < \phi < 1$ measures intertemporal knowledge spillovers (standing-on-shoulders effect) and $0 < \lambda < 1$ measures returns to R&D effort (stepping-on-toes effect). $\bar{A}_t$ is the world technology frontier that is assumed to grow exogenously at rate, $g_{\bar{A}}$. The standing-on-shoulders effect may arise as existing knowledge contributes to the capacity to innovate. The returns to human capital differ between the firm level and the economy-wide level. There exists constant returns to R&D effort at the firm level as revealed by eq. (24). However, the R&D technology shows diminishing returns to R&D effort as researchers generate negative externality at the aggregate level (stepping-on-toes effect). The stepping-on-toes effect may arise due to competition among multiple R&D firms to become the first to succeed at creating and patenting a new blueprint and/or process. If all other factors are held constant, an increase in R&D effort will induce increased duplication of research efforts leading to stepping-on-toes effect. Additionally, R&D productivity depends on a catch-up term, $\frac{\bar{A}_t}{A_t}$ under imitation regime. Akin to Nelson and Phelps (1966), $\frac{\bar{A}_t}{A_t}$ is the catch-up term which signifies the fact that greater the technological gap between leader and follower economy, higher the potential of the follower economy to catch up through imitation of existing technologies. Since all R&D firms end up in a symmetric equilibrium, the production function of technology under imitation regime at the aggregate level reduces to:

$$A_{t+1} - A_t = \bar{\delta} H_t^{\lambda} A_t^{\phi} \left[\frac{\bar{A}_t}{A_t} \right]. \tag{27}$$

The catch-up effect is specific to imitation regime alone. Under the innovation regime, firms innovate upon the local technology level to discover new blueprints. In this case, the aggregate production function reduces to:

$$A_{t+1} - A_t = \bar{\delta} H_t^{\lambda} A_t^{\phi}. \tag{28}$$

Firms in the R&D sector maximize their profits, given by:

$$\pi_{t,A} = p_t^A (A_{t+1} - A_t) - w_t H_t, \tag{29}$$

where p_t^A is price of a blueprint, $A_{t+1} - A_t$ are number of new blueprints discovered and w_t is the wage rate.

Under both imitation and innovation regimes, using eq. (24), the profit function of an R&D firm can be expressed as:

$$\pi_{t,A} = p_t^A (\delta_t H_t) - w_t H_t. \tag{30}$$

Again, under both the technology regimes, maximization of profits leads to the following optimality condition:

$$w_t = p_t^A \delta_t. \tag{31}$$

Substituting for δ_t from eq. (25), the wage rate under innovation regime is given by:

$$w_t^{in} = p_t^A \bar{\delta} H_t^{\lambda - 1} A_t^{\phi} = \left[\frac{p_t^A \bar{\delta} H_t^{\lambda} A_t^{\phi}}{H_t} \right]. \tag{32}$$

Similarly, wage rate under imitation regime is given by:

$$w_t^{im} = \frac{p_t^A \bar{\delta} H_t^{\lambda} A_t^{\phi} \frac{\bar{A}_t}{A_t}}{H_t}. \tag{33}$$

Using eqs. (27) and (28), the wage rate under both the regimes simplifies to

$$w_t = \left[\frac{p_t^A(A_{t+1} - A_t)}{H_t}\right], \tag{34}$$

where wages of scientists are increasing in price of blueprint (price of patent) and number of blueprints discovered.

The decision to produce an intermediate variety by an intermediate input producer depends on the difference between the cost of acquiring the patent for a blueprint from the R&D sector, p_t^A, and the monopoly profits, π_t, that can be earned by producing intermediate varieties. Given this information, the R&D sector will set the price of patent, p_t^A such that it extracts the present discounted value of monopoly profits of intermediate firms. The research arbitrage argument goes as follows.

Suppose an intermediate firm faces two options. First, it can make an investment of p_t^A in physical capital and earn the market rate of interest, r_t. Alternatively, it can purchase a patent, earn profits in one period and, then, sell the patent. In equilibirum, the rate of return from both these investments should be the same. That is,

$$r_t p_t^A = \pi_t + p_{t+1}^A - p_t^A.$$

The l.h.s of this equation is the interest earned from investing in physical capital. The r.h.s is the sum of the profits earned and the capital gain/loss resulting from the change in price of patents over time. Rearranging the above equation yields the following research arbitrage condition:

$$r_t = \frac{\pi_t}{p_t^A} + \left[\frac{p_{t+1}^A - p_t^A}{p_t^A}\right]. \tag{35}$$

This equation states that R&D sector charges a price of blueprint, p_t^A, such that intermediate input producers are indifferent between purchasing a blueprint to produce an intermediate variety and not producing the intermediate variety at all. The dividend rate, given by, $\frac{\pi_t}{p_t^A}$. and the capital gain/loss, $\frac{p_{t+1}^A - p_t^A}{p_t^A}$, equal the market rate of return on investment, r_t. This research arbitrage condition yields the following price of blueprint:[2]

$$p_t^A = \frac{\pi_t}{1 + r_t - \left[\frac{(1+g_{K,t})}{(1+g_{A,t})}\right]^{\alpha}}. \tag{36}$$

[2]Detailed derivation is provided in Appendix B.

Inserting this in eq. (34), the wage rate under both the technology regimes can be expressed as:

$$w_t = \frac{\alpha(1-\alpha)}{1 + r_t - \left[\frac{(1+g_{K,t})}{(1+g_{A,t})}\right]^\alpha} \frac{Y_t}{H_t} g_{A,t}, \tag{37}$$

where $g_{K,t} = \frac{K_{t+1} - K_t}{K_t}$ and $g_{A,t} = \frac{A_{t+1} - A_t}{A_t}$.

Alternatively, similar to Strulik et al. (2013) and Prettner (2012), it can be assumed that patent protection for a newly discovered blueprint lasts only for one period t (i.e. one generation) to do away with issues of discounted present value of benefits from R&D. This leads to a slightly modified Romer (1990) production structure where patents last for one period. This assumption simplifies the exposition considerably as it keeps the basic incentive to create new knowledge intact while avoiding the intertemporal problems of patent pricing and patent holding. In the R&D sector, once a blueprint has been produced, a large number of potential intermediate input producers bid for the patent of the blueprint. The decision to produce a new intermediate variety depends on a comparison of operating profits that can be earned by producing an intermediate variety in time period t (when patent protection is valid) and the cost of buying blueprint. Since the market for blueprints is competitive, price of blueprint will be bid up until it is equal to the operating profit of intermediate input firm in period t. Therefore, price of blueprints can be written as:

$$p_t^A = \pi_t = \alpha(1-\alpha)\frac{Y_t}{A_t}, \tag{38}$$

which follows from eq. (21). Accordingly, wage rate in eq. (34) under both the regimes can be expressed as:

$$w_t = \alpha(1-\alpha)\frac{Y_t}{H_t} g_{A,t}, \tag{39}$$

where $g_{A,t} = \frac{A_{t+1} - A_t}{A_t}$.

3 Dynamics and Steady-State Properties of the Stylized Economy

3.1 Dynamics of the Key Variables

This section examines the dynamic properties of our stylized economy. First, we discuss the dynamics of physical factors of production. The aggregate population, N_t, grows at the fertility rate, n_t as follows:

$$N_{t+1} = n_t N_t, \tag{40}$$

where n_t is endogenously given by eq. (8). Taking child rearing time into account, the size of the workforce is given by $L_t = (1 - \tau n_t) N_t$. Since child rearing costs are constant, and from eq. (8) we know that fertility rate is also constant over time, the workforce grows at the fertility rate, as:

$$L_{t+1} = n_t L_t. \tag{41}$$

Assuming that physical capital depreciates fully within a generation (that is, depreciation is 100 percent) so that next period's capital stock consists of this period's aggregate savings, the market clearing condition for capital market will be

$$K_{t+1} = s_t N_t, \tag{42}$$

where N_t is the population of generation t.

If patents are infinitely-lived, then wage rate is given by eq. (37). Inserting the solutions for savings from eq. (6) and wage rate from eq. (37) and inserting eq. (20) into eq. (42) and using the fact that $N_t = L_t$ from eqs. (40) and (41), we get the equation governing the evolution of aggregate physical capital as:

$$K_{t+1} = B_1 K_t^{\alpha} A_t^{1-\alpha} g_{A,t}, \tag{43}$$

where $B_1 = \dfrac{\beta_1 \alpha (1-\alpha) l_t^{1-\alpha}}{(1+\beta_1+\beta_2)\left[1 + r_t - \left[\dfrac{(1+g_{K,t})}{(1+g_{A,t})}\right]^{\alpha}\right]}$.

For the alternate case, when patents last for one period, the equation for physical capital accumulation is expressed as:

$$K_{t+1} = B_2 K_t^{\alpha} A_t^{1-\alpha} g_{A,t}, \tag{44}$$

where $B_2 = \left[\frac{\beta_1}{1+\beta_1+\beta_2}\right] \alpha(1-\alpha) l_t^{1-\alpha}$. The above expression is derived after inserting the solutions for savings from eq. (6) and wage rate from eq.(39) and inserting eq. (20) into eq. (42) and using the fact that $N_t = L_t$ from eqs. (40) and (41).

Next, we discuss the dynamics of aggregate human capital, $H_t \equiv h_t L_t$. The dynamics of per capita human capital are given by eq. (9). Using eqs. (9) and (41), the equation for aggregate human capital accumulation can be written as:

$$\frac{H_{t+1}}{H_t} = \begin{cases} \mu^{\epsilon} n_t, & \text{if } \theta \leq \frac{\mu}{\tau\epsilon}; \\ \left[\dfrac{\epsilon(\tau\theta - \mu)}{(1-\epsilon)}\right]^{\epsilon} n_t, & \text{otherwise.} \end{cases} \tag{45}$$

From eqs. (27) and (28), the dynamics of total factor productivity can be expressed as:
Imitation regime:

$$A_{t+1} = A_t + \bar{\delta} H_t^{\lambda} A_t^{\phi} \left[\frac{\bar{A}_t}{A_t}\right], \tag{46}$$

Innovation regime:

$$A_{t+1} = A_t + \bar{\delta} H_t^{\lambda} A_t^{\phi}. \tag{47}$$

This system of equations fully describes the equilibrium dynamics of our model economy for all the plausible cases.The next subsection characterises the balanced growth paths of an economy for two cases - a) when an economy's quality of education system is sufficiently high, $\theta > \dfrac{\mu}{\tau\epsilon}$, and b) when quality of schooling is less than the threshold, $\theta \leq \dfrac{\mu}{\tau\epsilon}$.

3.2 Characterizing the Balanced Growth Path

A balanced growth path (BGP) is a long run equilibrium of the economy, also defined as the steady state, along which growth rate of variables is either zero or constant over time. For any variable x, the growth rate is denoted by $g_{x,t} = (x_{t+1} - x_t)/x_t$, and its rate of change by $\tilde{g}_{x,t} = (g_{x,t+1} - g_{x,t})/g_{x,t}$. The balanced growth, thus, requires $\tilde{g}_{x,t} = 0$. We denote the growth rate of x along the BGP by g_x, i.e., by omitting the time index for brevity. We begin by evaluating the physical capital accumulation along the BGP. When patents last forever, the growth rate of physical is derived from eq. (43) as:

$$1 + g_{K,t} \equiv \frac{K_{t+1}}{K_t} = \left[\frac{K_t}{K_{t-1}}\right]^{\alpha} \left[\frac{A_t}{A_{t-1}}\right]^{1-\alpha} \left[\frac{g_{A,t}}{g_{A,t-1}}\right] \left[\frac{1 + r_{t-1} - \left[\frac{(1+g_{K,t-1})}{(1+g_{A,t-1})}\right]^{\alpha}}{1 + r_t - \left[\frac{(1+g_{K,t})}{(1+g_{A,t})}\right]^{\alpha}}\right]. \tag{48}$$

When patents last for one period, we deduce from eq. (44) that:

$$1 + g_{K,t} \equiv \frac{K_{t+1}}{K_t} = \left[\frac{K_t}{K_{t-1}}\right]^{\alpha} \left[\frac{A_t}{A_{t-1}}\right]^{1-\alpha} \left[\frac{g_{A,t}}{g_{A,t-1}}\right]. \tag{49}$$

It has been proved in Appendix C that both the cases of infinitely-lived patents and one-period lasting patents yield the same steady state condition:

$$g_K = g_A. \tag{50}$$

The growth of physical capital and productivity are positively correlated along the steady state. Next, we consider the growth rate of total factor productivity, for which we observe from eq. (46) that rate of technical progress under the imitation regime can be written as:

$$1 + g_{A,t} \equiv \frac{A_{t+1}}{A_t} = 1 + \frac{\bar{\delta}^{\frac{1}{2-\phi}} H_t^{\frac{\lambda}{2-\phi}} \bar{A}_t^{\frac{1}{2-\phi}}}{A_t}. \tag{51}$$

Since along BGP, l.h.s is constant. Therefore, r.h.s must be constant as well, and this happens when

$$(1 + g_A) = (1 + g_H)^{\frac{\lambda}{2-\phi}} (1 + g_{\bar{A}})^{\frac{1}{2-\phi}} = [(1 + g_h)n]^{\frac{\lambda}{2-\phi}} (1 + g_{\bar{A}})^{\frac{1}{2-\phi}}. \tag{52}$$

The r.h.s follows from the definition of aggregate human capital, that is, $H_t = h_t L_t$ and eq. (41).[3]

Similarly, under innovation regime, we observe from eq. (47) that:

$$1 + g_{A,t} \equiv \frac{A_{t+1}}{A_t} = 1 + \frac{\bar{\delta}^{\frac{1}{1-\phi}} H_t^{\frac{\lambda}{1-\phi}}}{A_t}. \tag{53}$$

Using the definition of BGP, we derive the long run rate of technological progress under the innovation regime to be:

$$(1 + g_A) = [(1 + g_h)n]^{\frac{\lambda}{1-\phi}}. \tag{54}$$

[3]We have dropped the time index of fertility rate as fertility rate remains constant over time.

Thus, under both imitation and innovation regimes, technological progress is driven by growth in aggregate human capital. Human capital accumulation improves productivity of researchers, which fosters technological progress. Besides aggregate human capital, the growth of world technology frontier is an additional driver of growth under the imitation regime. The follower economy takes advantage of existing technologies through technology adoption. Therefore, as the world technology frontier grows, it enhances the potential of follower country to catch up through imitation.

Next, we ascertain the growth rates of aggregate GDP and per capita consumption along BGP. From eq. (20), we observe that

$$1 + g_{Y,t} \equiv \frac{Y_{t+1}}{Y_t} = \left(\frac{K_{t+1}}{K_t}\right)^{\alpha} \left(\frac{A_{t+1}}{A_t}\right)^{1-\alpha}; \tag{55}$$

Using eq. (50), the long run growth rate of GDP can be expressed as

$$g_Y = g_A. \tag{56}$$

Putting together all information from eqs. (50), (52), (54) and (56), we derive the balanced growth path of the economy under the two technology regimes.
Imitation regime:

$$g_K = g_Y = g_A = [(1+g_h)n]^{\frac{\lambda}{2-\phi}}(1+g_{\bar{A}})^{\frac{1}{2-\phi}} - 1; \tag{57}$$

Innovation regime:

$$g_K = g_Y = g_A = [(1+g_h)n]^{\frac{\lambda}{1-\phi}} - 1, \tag{58}$$

where

$$[(1+g_h)n] = (1+g_H) = \begin{cases} \dfrac{\beta_2\theta\epsilon}{(1+\beta_1+\beta_2)\mu^{1-\epsilon}}, & \text{if } \theta \leq \frac{\mu}{\tau\epsilon}; \\ \dfrac{\beta_2\theta\epsilon^{\epsilon}(1-\epsilon)^{1-\epsilon}}{(1+\beta_1+\beta_2)(\tau\theta-\mu)^{1-\epsilon}}, & \text{otherwise.} \end{cases}$$

This follows after substituting the value of n from eq. (8) in eq. (45). Furthermore, we observe from the consumer's optimisation exercise that:

$$\frac{c_{t+1}}{c_t} = \beta_1(1+r_{t+1}). \tag{59}$$

The r.h.s follows from substituting values of c_t and s_t from eqs. (5) and (6) in eq. (3). Using eqs. (23), (57) and (58), we derive that:

$$\frac{c_{t+1}}{c_t} = \beta_1 \left[1 + \alpha^2 \frac{Y_t}{K_t}\right]. \tag{60}$$

Along the BGP, since $g_K = g_Y$, per capita consumption grows at a constant rate under both the technology regimes.

We next analyze the evolution of wage rate and rate of interest along the steady state. From eq. (23), under both the technology regimes, rate of interest is given by:

$$r_t = \alpha^2 \frac{Y_t}{K_t}.$$

A closer examination of the above expression for rate of interest reveals that it will be constant along the BGP if A_t and K_t grow at the same rate. From eqs. (57) and (58), it is derived that along the BGP, this is indeed the case. That is,

$$g_K = g_Y.$$

This implies rate of interest is constant along the BGP under both the technology regimes, which is as one would expect. Further, when patents are infinitely-lived, we know that eq. (37) yields the following expression for wage rate under both the technology regimes:

$$w_t = \frac{\alpha(1-\alpha)}{1 + r_t - \left[\frac{1+g_{K,t}}{1+g_{A,t}}\right]^{\alpha}} \frac{Y_t}{H_t} g_{A,t},$$

where $g_{K,t} = \dfrac{K_{t+1} - K_t}{K_t}$ and $g_{A,t} = \dfrac{A_{t+1} - A_t}{A_t}$.

Alternatively, when patents last for one time-period, we know from eq. (39) that wage rate is expressed as

$$w_t = \alpha(1-\alpha)\frac{Y_t}{H_t} g_{A,t}.$$

From eq. (58), we know that under innovation regime:

$$g_K = g_Y = g_A = [(1+g_h)n]^{\frac{\lambda}{1-\phi}} - 1.$$

This implies that along the BGP for both the cases of finitely and infinitely-lived patents, wage rate is constant if aggregate human capital and aggregate output grow at the same rate. This holds true under the condition:

$$\lambda + \phi = 1. \tag{61}$$

The intuition behind this condition is that joint sum of returns to R&D effort and intertemporal knowledge spillovers should be equal to 1 to ensure constant returns to R&D sector. Only when this condition is met, aggregate human capital and aggregate output grow at the same rate and wage rate stabilizes.

Analogously, we know from eq. (57) that under imitation regime:

$$g_K = g_Y = g_A = [(1 + g_h)n]^{\frac{\lambda}{2-\phi}}(1 + g_{\bar{A}})^{\frac{1}{2-\phi}} - 1.$$

This implies that wage rate is constant along the BGP for both the cases of finitely and infinitely-lived patents if aggregate human capital and aggregate output grow at the same rate. This holds true only if

$$\lambda + \phi = 1 \quad \text{and} \quad (1 + g_{\bar{A}}) = (1 + g_H). \tag{62}$$

Thus, the parametric restriction in eq. (61) is necessary for the economy to be in steady state.

In what follows, we analyze what happens to the stylized economy when the parametric conditions in eq. (61) is not satisfied. Ideally, we should characterize the out-of-steady state dynamics of the macro-variables under the two technology regimes. However, out-of-steady state analysis is tedious as the calculations are mathematically intractable. Therefore, we discuss the behavior of the variables under the two regimes of technological improvement when the parametric condition specified in eq. (61) is not met around the BGP. Accordingly, it is assumed that the other parametric condition, $(1 + g_{\bar{A}}) = (1 + g_H)$ still holds as the economy is initially moving along the BGP.

Suppose the economy is on the BGP but due to some exogenous occurence, the value of λ or ϕ falls such that $\lambda+\phi < 1$. It has been proved in Appendix D that when $\lambda+\phi < 1$, the wage rate falls over time for both the cases of one-period lived and infinitely-lived patents under both the regimes of technological improvement. Consequently, per capita consumption and savings rate in absolute terms decline under both the technology regimes. Thus, both innovation and imitation economies will shrink over time if diminishing returns to R&D sector set in around the steady state, and the individual economies will diverge.

Alternatively, if the value of λ or ϕ rises, such that $\lambda + \phi > 1$ along the BGP, then it is shown again in Appendix D that under both the technology regimes, per capita consumption and savings rate rise in absolute terms as the wage rate rises over time for both the cases of infinitely-lived and one-period lived patents. However, this is only a theoretical possibility as there is no empirical support for the hypothesis that there exists increasing returns to R&D sector, that is, $\lambda + \phi > 1$ in the real world. Thus, an economy on the BGP implies that

$$\lambda + \phi = 1,$$

which is, therefore, a necessary condition for the steady state under both the innovation and imitation regimes.

Further, from eq. (57), $(1 + g_H)$ under imitation regime can be expressed as:

$$(1 + g_H) = \frac{(1 + g_A)^{\frac{2-\phi}{\lambda}}}{(1 + g_{\bar{A}})^{\lambda}}, \tag{63}$$

Substituting in eq. (62) and simplifying for $(1 + g_{\bar{A}})$, we get that

$$(1 + g_{\bar{A}}) = (1 + g_A)^{\frac{2-\phi}{\lambda(1+\lambda)}}. \tag{64}$$

Thus, the frontier economy will grow at the above rate along the BGP. Also, it can be observed that the frontier economy will grow at a higher rate than the follower economy only if

$$\frac{2 - \phi}{\lambda(1 + \lambda)} > 1, \tag{65}$$

which will be true if

$$\lambda + \phi < 2 - \lambda^2. \tag{66}$$

This is indeed the case given $\lambda + \phi = 1$ from eq. (62). Thus, the evolution of wage rate and rate of interest can be stated in terms of the following proposition.

Proposition 3.1 *In case of both one-period and infinitely-lived patents, when* $\theta > \frac{\mu}{\tau\epsilon}$ *or* $\theta \leq \frac{\mu}{\tau\epsilon}$*:*

- *Under both the innovation and imitation regimes, rate of interest is constant along the balanced growth path.*
- *Wage rate is constant under innovation regime along the balanced growth path implying the necessary condition:*

$$\lambda + \phi = 1.$$

- *Wage rate is constant under imitation regime along the balanced growth path under the necessary condition:*

$$g_{\bar{A}} = (1 + g_A)^{\frac{2-\phi}{\lambda(1+\lambda)}} \quad \textit{and} \quad \lambda + \phi = 1$$

Additionally, it can be deduced from eq. (58) that when $\lambda + \phi = 1$ holds, the BGP under innovation regime can be redefined as

$$g_K = g_Y = g_A = [(1 + g_h)n] - 1, \tag{67}$$

where

$$[(1 + g_h)n] = (1 + g_H) = \begin{cases} \dfrac{\beta_2 \theta \epsilon}{(1 + \beta_1 + \beta_2)\mu^{1-\epsilon}}, & \text{if } \theta \leq \frac{\mu}{\tau\epsilon}; \\ \dfrac{\beta_2 \theta \epsilon^{\epsilon}(1-\epsilon)^{1-\epsilon}}{(1 + \beta_1 + \beta_2)(\tau\theta - \mu)^{1-\epsilon}}, & \text{otherwise.} \end{cases}$$

Furthermore, we derive the conditions when the economic growth rate is higher for an economy with high quality of schooling, $\theta > \dfrac{\mu}{\tau\epsilon}$ as compared to an economy with lower quality of schooling, $\theta \leq \dfrac{\mu}{\tau\epsilon}$ under the two technology regimes. We assume that when $\theta > \dfrac{\mu}{\tau\epsilon}$, quality of schooling is denoted by θ_h for that particular economy whereas quality of schooling is denoted by θ_l for an economy with quality of schooling less than the threshold, $\theta \leq \dfrac{\mu}{\tau\epsilon}$. As shown in eqs. (67) and (57), the rate of economic growth is contingent upon the rate of human capital accumulation under the two technology regimes. This means that under the two technology regimes, an economy with higher quality of schooling, θ_h, grows at a higher rate as compared to an economy with a lower quality of schooling, θ_l when the following condition holds true:

$$g_{H|(\theta_h > \frac{\mu}{\tau\epsilon})} > g_{H|(\theta_l \leq \frac{\mu}{\tau\epsilon})}$$

Substituting for $g_{H|\theta_h > \frac{\mu}{\tau\epsilon}}$ and $g_{H|(\theta_l \leq \frac{\mu}{\tau\epsilon})}$ from eqs. (67) and (57), we have:

$$\frac{\beta_2 \theta_h \epsilon^{\epsilon}(1-\epsilon)^{1-\epsilon}}{(1+\beta_1+\beta_2)(\tau\theta_h-\mu)^{1-\epsilon}} > \frac{\beta_2 \theta_l \epsilon}{(1+\beta_1+\beta_2)\mu^{1-\epsilon}},$$

which on simplification yields the following condition:

$$\theta_h > \theta_l \left[\frac{(\tau\theta_h-\mu)\epsilon}{\mu(1-\epsilon)}\right]^{1-\epsilon}. \tag{68}$$

We know that,

$$\theta > \frac{\mu}{\tau\epsilon}.$$

Multiplying both sides by τ and then, subtracting μ from both sides yields

$$\frac{(\tau\theta-\mu)\epsilon}{1-\epsilon} > \mu,$$

Since $\mu \geq 1$, we have:

$$\frac{(\tau\theta-\mu)\epsilon}{1-\epsilon} > 1. \tag{69}$$

Thus, $\left[\frac{(\tau\theta_h-\mu)\epsilon}{\mu(1-\epsilon)}\right]^{1-\epsilon} > 1$. Thus, it can be deduced that,

Proposition 3.2 *When the necessary conditions stated in Proposition 3.1 are satisfied, then*

- *Under innovation regime, aggregate output, physical capital stock, total factor productivity and per capita consumption grow at a constant rate along the balanced growth path characterized by eqs. (67) and (60).*
- *Under imitation regime, aggregate output, physical capital stock, total factor productivity and per capita consumption grow at a constant rate along the balanced growth path characterized by eqs. (57) and (60).*
- *Under the two technology regimes, an economy with higher quality of schooling, $\theta_h > \frac{\mu}{\tau\epsilon}$, experiences a higher economic growth as compared to an economy with lower quality of schooling, $\theta_l \leq \frac{\mu}{\tau\epsilon}$ only if the quality of schooling, θ_h is sufficiently high such that:*

$$\theta_h > \theta_l \left[\frac{(\tau\theta_h-\mu)\epsilon}{\mu(1-\epsilon)}\right]^{1-\epsilon}. \tag{70}$$

Intuitively, under both the technology regimes, the self-sustaining growth path is driven by human capital accumulation when quality of schooling exceeds the threshold, $\theta > \frac{\mu}{\tau\epsilon}$. At the micro level, parents decide to have fewer number of children and invest more in their education. This follows from Lemma 1. At the macro level, this trade-off raises the rate of human capital accumulation, which encourages faster technological progress and, therefore, economic growth. Besides human capital, growth of world technology frontier is an additional driver of growth under the imitation regime via the catch-up effect.

Alternatively, when quality of schooling is less than the threshold, $\theta \leq \frac{\mu}{\tau\epsilon}$, parents do not invest in education of children and instead maximize fertility. In this case, the balanced growth path of the economy is driven only by population growth, which in turn, is determined by the fertility rate. Thus, the drivers of economic growth differ depending upon the level of quality of schooling. When quality of schooling surpasses the threshold level, economic growth is driven by human capital accumulation whereas it is driven by population growth when quality of schooling is less than the threshold.

Furthermore, a mere surpassing of the threshold level of quality schooling is not sufficient enough for an economy to experience a higher economic growth rate as compared to an economy with quality of schooling lower than the threshold level. Under the two technology regimes, quality of schooling should be high enough such that it leads to high enough investments in education of children such that the growth-stimulating effect overpowers the growth-impeding effect of quality of schooling by a larger magnitude. This, in turn, can only ensure that an economy with a higher quality of schooling (that is, $\theta_h > \dfrac{\mu}{\tau\epsilon}$) experiences a higher economic growth as compared to an economy with a lower quality of schooling (that is, $\theta_l \leq \dfrac{\mu}{\tau\epsilon}$).

Otherwise, an economy with lower quality of schooling may experience a higher economic growth rate than an economy with higher quality of schooling for large enough values of child rearing costs, τ or for small enough value of intergenerational human capital spillovers, μ and returns to education, ϵ respectively. This follows directly from eq. (70). It can be observed that the expression, $\dfrac{(\tau\theta_h - \mu)\epsilon}{\mu(1-\epsilon)}$ is increasing in τ. This implies that the threshold value of quality of schooling for higher economic growth is so high that an economy may consider not investing an education of the future generation as a relatively more beneficial outcome. In this particular case when the value of τ is sufficiently high, population growth rate can be a more effective driver of economic growth . Similarly, it can be shown that:

$$\frac{\partial}{\partial \mu}\frac{(\tau\theta_h - \mu)}{\mu} = \frac{-\tau\theta_h}{\mu^2} < 0;$$
$$\frac{\partial}{\partial \epsilon}\frac{\epsilon}{1-\epsilon} = \frac{-1}{(1-\epsilon)^2} < 0.$$

This implies that the threshold value of quality of schooling for higher economic growth is decreasing in the value of μ and ϵ respectively. Thus, this threshold value of quality of schooling can be high enough for sufficiently small μ and ϵ such that population growth rate can be a more effective driver of economic growth and an economy may not invest in human capital of its future generation.

Additionally, eqs. (57) and (67) suggest that technological progress and aggregate output are positively correlated with population growth. This implies that decline in population growth entails a decline in rate of technical progress as postulated by conventional R&D based growth models (Jones, 1995; Romer, 1990). This type of macro-level superficial examination misses the point that aggregate human capital accumulation and fertility rate are inversely related via quality-quantity trade-off at the family/household level as shown in Lemma 1 and 2. The investment in education increases and fertility rate falls simultaneously as the quality of schooling increases above the threshold. This quality-quantity trade-off implies that the effect of population growth on total factor productivity growth and GDP growth cannot be analyzed in isolation keeping human capital growth constant. This leads to the question: how does improvement in quality of schooling and returns to education affect total factor productivity growth and, therefore, economic growth by influencing fertility and education decisions?

This can be answered by carrying out comparative dynamics with respect to these parameters that has been done in the next subsection.

3.3 Comparative Dynamics Analyses of the Balanced Growth Path

3.3.1 Comparative Dynamics w.r.t Quality of Schooling, θ

Since total factor productivity growth and economic growth depend on the rate of human capital accumulation under both the regimes of technological improvement, we first carry out comparative dynamics of the growth rate of aggregate human capital with respect to schooling quality. Let the economy be in steady state now. We take the derivative of the growth rate of aggregate human capital with respect to schooling quality, θ, when it exceeds the threshold, that is, $\theta > \frac{\mu}{\tau\epsilon}$. Detailed derivations of eqs. (71) and (72) are provided in Appendix E.

$$\frac{\partial g_H}{\partial \theta} = \left[\frac{(1+g_h)\beta_2(\epsilon\tau\theta - \mu)(1-\epsilon)}{(1+\beta_1+\beta_2)(\tau\theta-\mu)^2} \right] > 0, \tag{71}$$

in view of $\epsilon < 1$. Thus, in the aggregate, the growth rate of technology increases in response to an increase in schooling quality which sustains economic growth in the long-run.

We next analyze the derivative of the growth rate of aggregate human capital with respect to schooling quality when it is less than the threshold, that is, $\theta \leq \frac{\mu}{\tau\epsilon}$. We get that,

$$\frac{\partial g_H}{\partial \theta} = \left[\frac{\beta_2 \epsilon}{(1+\beta_1+\beta_2)\mu^{1-\epsilon}} \right] > 0. \tag{72}$$

Thus, it can be deduced that,

Proposition 3.3 *The long-run rate of technical progress, g_A, and economic growth, g_Y, increase in response to an improvement in the quality of schooling on account of different channels, depending upon the quality of schooling, θ:*

- *When $\theta > \frac{\mu}{\tau\epsilon}$, g_A and g_Y are increasing in θ due to higher rate of human capital accumulation under both the regimes of technological improvement.*
- *When $\theta \leq \frac{\mu}{\tau\epsilon}$, g_A and g_Y are increasing in θ due to higher population growth rate under both the regimes of technological improvement .*

The intuitive explanation for the impact of a change in quality of schooling on the long-run rate of technical progress and economic growth is as follows. When quality of schooling surpasses the threshold, it has two opposing effects on human capital accumulation. We know from Lemma 1 that an improvement in quality of schooling increases investment in the education of a child. This stimulates the accumulation of human capital which fosters technical progress leading to higher economic growth in the economy. This effect can be regarded as the growth-stimulating effect. The increase in education is also accompanied by a decline in fertility rate as the quality of education improves. This constitutes the growth-impeding effect that reduces the total factor productivity growth and economic growth by contracting the pool of available researchers. Total factor productivity growth and economic growth will accelerate or decelerate depending upon the relative magnitude of the two effects. As shown by eq. (71), the growth-stimulating effect overpowers the growth-impeding effect of a change in quality of schooling when quality of schooling exceeds the threshold, that is, $\theta > \dfrac{\mu}{\tau\epsilon}$.

When quality of schooling is less than the threshold, parents do not educate their children and instead focus on having more children. In this particular case, there exist no growth-stimulating and growth-impeding effects of quality of schooling. In this case, the rate of technical progress and economic growth increase in response to an increase in the quality of schooling solely due to higher population growth, as parents focus on maximizing fertility when quality of schooling is less than the threshold.

Thus contingent upon the quality of schooling, there are two different channels at work which foster technical progress and economic growth. On one hand, quality of schooling raises total factor productivity growth and economic growth due to better quality of human capital in the economy as parents educate their children when quality of schooling surpasses the threshold. On the other hand, quality of schooling raises total factor productivity growth and economic growth due to higher population growth as parents have more children and impart no education to them when quality of schooling is less than the threshold. This result is similar to Hashimoto and Tabata (2016) finding about old-age survival probability and economic growth. They find that in economies in which old-age survival probability is sufficiently low, an increase in old-age survival probability motivates individuals to invest more in their own education, accelerating the accumulation of per capita human capital and, thereby, enhancing the long-run growth rate of the economy. However, in economies where old-age survival probability is sufficiently high, an increase in old age survival probability will lead to decline in population growth rates, thereby lowering the long-run growth rate of the economy.

We next consider the comparative dynamics of per capita income, $y_t = Y_t/L_t$ with respect to θ. At the steady state, the growth rate of per capita income under the innovation regime is given by:[4]

$$(1 + g_y) = (1 + g_h). \tag{73}$$

[4]Detailed derivation is provided in Appendix F

And under the imitation regime, it is given by:

$$g_y = (1 + g_{\bar{A}})^{\frac{1}{2-\phi}}(1 + g_h)^{\frac{\lambda}{2-\phi}} n^{\frac{-1}{2-\phi}} - 1. \tag{74}$$

When $\theta > \frac{\mu}{\tau\epsilon}$, differentiating per capita income growth rate with respect to θ under the innovation regime yields:

$$\frac{\partial g_y}{\partial \theta} = \frac{(1 + g_y)\tau\epsilon}{\tau\theta - \mu} > 0, \tag{75}$$

since $\theta > \frac{\mu}{\tau\epsilon} \Rightarrow \theta > \frac{\mu}{\tau}$ as $\epsilon < 1$.

Similarly, if we differentiate per capita income growth rate with respect to θ under the imitation regime, we get that:

$$\frac{\partial g_y}{\partial \theta} = \frac{1 + g_y}{2 - \phi}\left[\frac{\lambda\tau\epsilon}{\tau\theta - \mu} + \frac{\mu}{\theta(\tau\theta - \mu)}\right] > 0, \tag{76}$$

as $\theta > \frac{\mu}{\tau\epsilon}$ and $\epsilon < 1$.

We next determine the comparative dynamics of g_y with respect to θ when $\theta \leq \frac{\mu}{\tau\epsilon}$. Differentiating growth rate of per capita income with respect to θ yields[5]-

Innovation regime:

$$\frac{\partial g_y}{\partial \theta} = 0; \tag{77}$$

Imitation regime:

$$\frac{\partial g_y}{\partial \theta} = \frac{-(1 + g_y)}{\theta(2 - \phi)} < 0. \tag{78}$$

An examination of these derivatives yields the following result.

Proposition 3.4 *Along the balanced growth path,*

- *When $\theta > \frac{\mu}{\tau\epsilon}$, g_y is unambiguously increasing in θ under both the innovation and imitation regimes of technological improvement.*
- *When $\theta \leq \frac{\mu}{\tau\epsilon}$, g_y is unaffected by θ under the innovation regime of technological improvement. However, g_y is unambiguously decreasing in θ under the imitation regime of technological improvement.*

[5]Detailed derivation of eqs. (77) and (78) are again provided in Appendix F.

The intuitive explanation for these results is as follows. We know that per capita income is $y_t = Y_t/L_t$. The growth rate of per capita income along the BGP can be expressed as $(1+g_y) = \frac{(1+g_Y)}{n}$. It is known from Proposition 3.3 that the economic growth rate, g_Y, is increasing in θ as growth-stimulating effect dominates the growth-impeding effect of quality of schooling when it exceeds the threshold. Also, it is known from Lemma 1 that parents bear lower number of children in response to an improvement in quality of schooling. Thus, the fertility rate or the population growth rate is decreasing in θ. Consequently, the growth rate of per capita income rises as quality of schooling improves under both the technology regimes when $\theta > \frac{\mu}{\tau\epsilon}$.

Alternatively, when $\theta \leq \frac{\mu}{\tau\epsilon}$, we know from eq. (9) that:

$$(1 + g_h) = \mu^{\epsilon}.$$

It follows from eq. (7) that parents do not invest in education of children when quality of schooling is less than the threshold and, therefore, human capital consists of basic skills only. As a result, quality of schooling has no impact on g_y under the innovation regime as along BGP:

$$g_y = (1 + g_h).$$

This follows from eq. (73).

Under the imitation regime, it follows from eqs. (74) and (9) that:

$$g_y = (1 + g_{\bar{A}})^{\frac{1}{2-\phi}}(\mu)^{\frac{\epsilon\lambda}{2-\phi}}n^{\frac{-1}{2-\phi}} - 1.$$

It has been shown in Lemma 1 that parents maximize fertility and do not spend on education of their children. Thus, fertility rate rises as quality of schooling rises when $\theta \leq \frac{\mu}{\tau\epsilon}$. As a consequence, growth rate of per capita income falls as quality of schooling rises.

Next, the comparative dynamics with respect to returns to education, ϵ, are discussed.

3.3.2 Comparative Dynamics w.r.t Returns to Education, ϵ

Since rate of human capital accumulation determines the rate of technical progress and economic growth, we first analyze the derivative of the growth rate of aggregate human capital with respect to returns to education. We get that,[6]

$$\frac{\partial g_H}{\partial \epsilon} = (1+g_H)log\left[\frac{(\tau\theta-\mu)\epsilon}{1-\epsilon}\right] > 0, \tag{79}$$

as $\dfrac{(\tau\theta-\mu)\epsilon}{1-\epsilon} > 1$ from eq. (69).

Similarly, when $\theta \leq \dfrac{\mu}{\tau\epsilon}$, the derivative of the growth rate of aggregate human capital with respect to returns to education yields:

$$\frac{\partial g_H}{\partial \epsilon} = (1+g_H)\left[\frac{1}{\epsilon}+log\mu\right] > 0. \tag{80}$$

Thus, we have,

Proposition 3.5 *The long-run rate of technical progress, g_A, and aggregate output, g_Y, increase in response to an increase in returns to education, ϵ, on account of different channels, depending upon the quality of schooling, θ:*

- *When $\theta > \dfrac{\mu}{\tau\epsilon}$, g_A and g_Y are increasing in ϵ due to higher rate of human capital accumulation under both the regimes of technological improvement.*
- *When $\theta \leq \dfrac{\mu}{\tau\epsilon}$, g_A and g_Y are increasing in ϵ due to higher intergenerational human capital spillovers and higher population growth rate.*

The intuitive explanation for the impact of a change in returns to education on the long-run rate of technical progress and economic growth is as follows. We know from Lemma 2 that an increase in returns to education triggers a child quantity-quality trade-off at the micro level. The threshold value of quality of schooling, $\dfrac{\mu}{\tau\epsilon}$, is decreasing in the value of ϵ. This implies that, ceteris paribus, this critical threshold value decreases as returns to schooling increase when quality of schooling exceeds the threshold. Therefore, parents educate their children and bear lower number of children in response to an increase in returns to education. Similar to the impact of quality of schooling, this micro level trade-off generates a growth-stimulating effect and a growth-impeding effect at the macro level. The growth-stimulating effect overpowers the growth-impeding effect of a change in returns to education when quality of schooling exceeds the threshold. Resultantly, an increase in returns to education yields higher rate of technical progress and, therefore, higher economic growth under both innovation and imitation regimes of technological improvement.

[6] Detailed derivations of eqs. (79) and (80) are provided in Appendix G.

Alternatively, when quality of schooling is less than the threshold, parents do not educate their children and, instead, focus on having more children. Therefore, similar to the effect of quality of schooling, the rate of technical progress increases in response to an increase in the returns to education due to higher population growth. Additionally, it can be observed from eq. (9) that intergenerational human capital spillovers become more productive and spur growth rate of per capita human capital as returns to education increase. On the whole, when $\theta \leq \frac{\mu}{\tau\epsilon}$, an increase in returns to education yield higher rate of technical progress and, therefore, economic growth under both innovation and imitation regimes due to higher growth rate of population and higher intergenerational human capital spillovers.

We next consider the impact of a change in returns to education, ϵ, on the growth rate of per capita income along the balanced growth path under the two technology regimes. When $\theta > \frac{\mu}{\tau\epsilon}$, differentiating per capita income growth rate with respect to ϵ under the innovation and imitation regimes, yields[7]-

Innovation regime:

$$\frac{\partial g_y}{\partial \epsilon} = (1 + g_y)\left[\frac{1}{1-\epsilon} + log\frac{(\tau\theta - \mu)\epsilon}{1-\epsilon}\right] > 0; \quad \text{and} \tag{81}$$

Imitation regime:

$$\frac{\partial g_y}{\partial \epsilon} = \frac{\lambda(1 + g_y)}{2-\phi}\left[\frac{1}{1-\epsilon} + log\frac{(\tau\theta - \mu)\epsilon}{1-\epsilon}\right] + \frac{(1+g_y)}{(2-\phi)(1-\epsilon)} > 0, \tag{82}$$

since $\epsilon < 1$ and $\frac{(\tau\theta - \mu)\epsilon}{1-\epsilon} > 1$ (from eq. (71) under both innovation and imitation regimes.

Further, we examine the case when $\theta \leq \frac{\mu}{\tau\epsilon}$. Differentiating per capita income growth rate with respect to ϵ yields -

Innovation regime:

$$\frac{\partial g_y}{\partial \epsilon} = (1 + g_y)log\mu > 0, \tag{83}$$

since $\epsilon < 1$ and $\mu \geq 1$.

Imitation regime:

$$\frac{\partial g_y}{\partial \epsilon} = \frac{\lambda(1 + g_y)log\mu}{2-\phi} - \frac{(1+g_y)}{(2-\phi)\epsilon}. \tag{84}$$

[7]Detailed derivation of eqs. (81), (82), (83) and (84) are provided in Appendix H

Now, $\frac{\partial g_y}{\partial \epsilon} < 0$ if $\lambda \epsilon log\mu < 1$ which is true as $\lambda < 1$, $\epsilon < 1$ and $\mu \geq 1$. An examination of these derivatives yields the following results.

Proposition 3.6 *Along the balanced growth path,*

- *When* $\theta > \frac{\mu}{\tau\epsilon}$, g_y *is unambiguously increasing in* ϵ *under both the innovation and imitation regimes of technological improvement.*
- *When* $\theta \leq \frac{\mu}{\tau\epsilon}$, g_y *is increasing in* ϵ *under the innovation regime of technological improvement. However,* g_y *is unambiguously decreasing in* ϵ *under the imitation regime of technological improvement.*

The intuitive explanation for this result is as follows. We know that per capita income is given by $y_t = Y_t/L_t$. Accordingly, the growth rate of per capita income along the BGP can be expressed as $(1 + g_y) = \frac{(1 + g_Y)}{n}$. It is known from Proposition 3.5 that the economic growth rate, g_Y, is increasing in ϵ as the growth-stimulating effect overpowers the growth-impeding effect of returns to education when quality of schooling exceeds the threshold. Also, it is known from Lemma 2 that parents bear a lower number of children in response to a rise in returns to education. Thus, akin to quality of schooling, the fertility rate or the population growth rate is decreasing in returns to education, ϵ. Consequently, the growth rate of per capita income rises as quality of schooling improves under both the technology regimes.

We next consider the case when $\theta \leq \frac{\mu}{\tau\epsilon}$. From eq. (73), it is known that along BGP, under the innovation regime:

$$g_y = g_h.$$

When $\theta \leq \frac{\mu}{\tau\epsilon}$, from eq. (9) we have that:

$$g_h = \mu^{\epsilon} - 1.$$

On the one hand, when $\mu = 1$, $g_h = 0$, returns to education has no impact on g_y under the innovation regime. On the other hand, when $\mu > 1$, it can be observed from eq. (9) that intergenerational human capital spillovers become more productive. Thus, growth rate of per capita human capital and, therefore, growth rate of per capita income increases as returns to education increase. Under the imitation regime, it follows from eq. (74) that:

$$g_y = (1 + g_{\bar{A}})^{\frac{1}{2-\phi}} (1 + g_h)^{\frac{\lambda}{2-\phi}} n^{\frac{-1}{2-\phi}} - 1.$$

It follows from Proposition 3.5 that g_Y is increasing in ϵ as intergenerational human capital spillovers become more productive and spur growth rate of per capita human capital as returns to education increase. Also, it is known from Lemma 2 that parents maximize fertility in response to a rise in returns to education when $\theta \leq \dfrac{\mu}{\tau\epsilon}$. It can be observed from eq. (74) that returns to education raise population growth rate by a larger proportion as compared to the proportionate rise in growth rate of per capita human capital as $\lambda < 1$. Therefore, growth rate of per capita income falls as returns to education improve under imitation regime of technological improvement.

This completes the characterization of the balanced growth path of our decentralised economy.

4 Discussion

The existing literature on quality of schooling and economic growth shows that economic growth and quality of schooling are positively correlated (see e.g. Bosworth & Collins, 2003; Ciccone & Papaioannou, 2009; Islam, Ang, & Madsen, 2014b). This implies that developing countries should adopt a two-pronged approach to enhance the skill set of its workers. Under this approach, countries should focus on improving quality of education also alongwith improving access to education. Motivated by these observations, this chapter formulates an analytical framework to analyze the impact of quality of schooling on economic growth of an economy under imitation and innovation regimes. An overlapping generations version of an R&D-based growth model (á la Diamond, 1965) and Jones (1995) is build to examine how improvement in quality of schooling impact technical progress and long-run economic growth of an economy by influencing fertility and education decisions at household level. We characterize two types of economies. First type of economy is an innovation economy where technological improvements occur by innovating on local technology frontier. The second type of economy is the imitation economy where technological progress occurs by imitating existing foreign technologies. We find that the quality of schooling triggers a child quantity-quality trade-off at the micro level when quality of schooling surpasses an endogenously determined threshold under both the regimes. When quality of schooling surpasses the threshold, then parents invest in education of their children and bear lower number of children. However, parents focus on maximizing fertility and do not educate their children when quality of schooling is less than the threshold. This micro-level trade-off has repercussions at the macro level. This micro-level trade-off has two opposing effects on aggregate human capital accumulation at macro level. Higher investment in education of a child stimulates the accumulation of human capital which fosters technical progress but the simultaneous decline in fertility rate reduces the total factor productivity growth and economic growth by contracting the pool of available researchers. The first effect prevails over latter only when quality of schooling is higher than the threshold. Accordingly, the economy is on a self-sustaining growth path propeled by higher human capital accumulation in the long-run when quality of schooling is higher than the threshold. When quality of schooling is less than the threshold, then parents do not educate their children and focus on maximizing fertility. In such a scenario, higher fertility rate leads to higher population growth which propels economic growth rate under both innovation and imitation regimes.

Further, the comparative dynamics analysis reveal that when quality of schooling exceeds the threshold, the long-run rate of technical progress and economic growth are increasing in quality of schooling and returns to education due to higher rate of human capital accumulation under both the regimes of technological improvement. Additionally, growth rate of per capita income is unambiguously increasing in quality of schooling and returns to education under both the technology regimes. When quality of schooling is less than the threshold, the long-run rate of technical progress and economic growth are increasing in quality of schooling and returns to education due to higher rate of population growth under both the regimes of technological improvement. Besides population growth, higher intergenerational human capital spillovers are another factor that propels economic growth when returns to education increase. Under the innovation regime, the growth rate of per capita income is unaffected by quality of schooling whereas it is increasing in returns to education when quality of schooling is less than the threshold. Under imitation regime, growth rate of per capita income is unambiguously decreasing in quality of schooling and returns to education.

A Solution to Household's Optimization Exercise

The utility function is described as follows:

Maximize

$$u_t = \log c_{1,t} + \beta_1 \log c_{2,t+1} + \beta_2 \log(h_{t+1} n_t)$$

$$\text{subject to}$$

$$w_t h_t (1 - \tau n_t) = c_{1,t} + s_t + e_t(w_t h_t) n_t$$

$$c_{2,t+1} = (1 + r_{t+1}) s_t$$

$$h_{t+1} = (\mu + \theta e_t)^{\epsilon} h_t, \qquad \epsilon < 1$$

After substituting for $c_{2,t+1}$ and h_{t+1}, the langragean for this problem is formulated as :

$$L = \log c_{1,t} + \beta_1 \log[(1 + r_{t+1}) s_t] + \beta_2 \log n_t + \beta_2 \epsilon \log(\mu + \theta e_t) + \beta_2 \epsilon \log h_t$$
$$+ \psi[w_t h_t (1 - \tau n_t) - c_{1,t} - s_t - e_t n_t (w_t h_t)]$$

The choice variables are $c_{1,t}$, s_t,e_t and n_t.The first-order conditions are:

$$\frac{\partial L}{\partial c_{1,t}} = 0 \Leftrightarrow \frac{1}{c_{1,t}} - \psi = 0 \Leftrightarrow c_{1,t} = \frac{1}{\psi}. \tag{85}$$

$$\frac{\partial L}{\partial s_t} = 0 \Leftrightarrow \frac{\beta_1}{s_t} - \psi = 0 \Leftrightarrow s_t = \frac{\beta_1}{\psi}. \tag{86}$$

$$\frac{\partial L}{\partial n_t} = 0 \Leftrightarrow \frac{\beta_2}{n_t} - \psi \tau w_t h_t - \psi e_t w_t h_t = 0 \Leftrightarrow \frac{\beta_2}{n_t} = \psi[\tau + e_t] w_t h_t \Leftrightarrow n_t = \frac{\beta_2}{\psi[\tau + e_t] w_t h_t}. \tag{87}$$

$$\frac{\partial L}{\partial e_t} = 0 \Leftrightarrow \frac{\beta_2 \epsilon \theta}{\mu + \theta e_t} - \psi n_t w_t h_t = 0 \Leftrightarrow n_t = \frac{\beta_2 \epsilon \theta}{\psi[\mu + \theta e_t] w_t h_t}. \tag{88}$$

From eqs. (87) and (88), the l.h.s can be equated to yield:

$$\mu + \theta e_t = \epsilon \theta[\tau + e_t] \Leftrightarrow \mu - \epsilon \theta \tau = e_t \theta[\epsilon - 1]$$

$$e_t = \frac{\mu - \epsilon \theta \tau}{\theta(\epsilon - 1)} = \frac{\epsilon \theta \tau - \mu}{\theta(1 - \epsilon)}$$

Hence, we have:

$$e_t = \begin{cases} 0, & \text{if } \theta \leq \frac{\mu}{\tau\epsilon}; \\ \dfrac{\tau\theta\epsilon - \mu}{\theta(1-\epsilon)}, & \text{otherwise.} \end{cases} \tag{89}$$

Next, we know that the budget constraint is given by:

$$w_t h_t(1 - \tau n_t) = c_{1,t} + s_t + e_t(w_t h_t)n_t.$$

From eq. (87), $e_t n_t(w_t h_t)$ can be expressed as:

$$e_t n_t(w_t h_t) = \frac{\beta_2}{\psi} - \tau n_t w_t h_t. \tag{90}$$

Substituting from eqs. (85), (86) and (90), the budget constraint can be expressed as:

$$w_t h_t - \tau n_t w_t h_t = \frac{1}{\psi} + \frac{\beta_1}{\psi} + \frac{\beta_2}{\psi} - \tau n_t w_t h_t$$

which on simplifying leads to:

$$\psi = \frac{1 + \beta_1 + \beta_2}{w_t h_t} \tag{91}$$

whose substitution into eqs. (85) and (86) yields:

$$c_{1,t} = \frac{w_t h_t}{1 + \beta_1 + \beta_2}; \tag{92}$$

$$s_t = \frac{\beta_1 w_t h_t}{1 + \beta_1 + \beta_2}, \tag{93}$$

Substituting for e_t from eq. (89) and for ψ from eq. (91) in eq. (87), yields:

$$n_t = \begin{cases} \dfrac{\beta_2 \epsilon\theta}{(1 + \beta_1 + \beta_2)\mu}, & \text{if } \theta \leq \frac{\mu}{\tau\epsilon}; \\ \dfrac{\beta_2\theta(1-\epsilon)}{(1 + \beta_1 + \beta_2)(\tau\theta - \mu)}, & \text{otherwise.} \end{cases} \tag{94}$$

This completes the solution to the utility maximization exercise of households.

B Derivation of Eq. (36)

As specified in eq. (35), the research arbitrage condition is given by:

$$r_t = \frac{\pi_t}{p_t^A} + \left[\frac{p_{t+1}^A - p_t^A}{p_t^A}\right].$$

We know from Proposition 3.1 that r_t is constant along balanced growth path. This implies l.h.s of above equation is constant. For r.h.s to be constant, following condition should hold true:

$$\frac{\pi_{t+1}}{\pi_t} = \frac{p^A_{t+1}}{p^A_t}$$

From eq. (21), we have:

$$\pi_t = \alpha(1-\alpha)\frac{Y_t}{A_t}.$$

This implies that:

$$\frac{\pi_{t+1}}{\pi_t} = \frac{\frac{Y_{t+1}}{Y_t}}{\frac{A_{t+1}}{A_t}}. \tag{95}$$

From eq. (20), we observe that:

$$1 + g_{Y,t} \equiv \frac{Y_{t+1}}{Y_t} = \frac{K_{t+1}}{K_t}^{\alpha} \frac{A_{t+1}}{A_t}^{1-\alpha}.$$

Inserting in eq. (95), we get that:

$$\frac{\pi_{t+1}}{\pi_t} = \left[\frac{\frac{K_{t+1}}{K_t}}{\frac{A_{t+1}}{A_t}}\right]^{\alpha} \equiv \left[\frac{1+g_{K,t}}{1+g_{A,t}}\right]^{\alpha}. \tag{96}$$

Thus, eq. (35) will hold true if

$$\frac{\pi_{t+1}}{\pi_t} = \frac{p^A_{t+1}}{p^A_t} = \left[\frac{1+g_{K,t}}{1+g_{A,t}}\right]^{\alpha}.$$

Inserting in eq. (35) and solving for p^A_t, we get that:

$$p^A_t = \frac{\pi_t}{1 + r_t - \left[\frac{(1+g_{K,t})}{(1+g_{A,t})}\right]^{\alpha}}.$$

C Derivation of Eq. (50)

When patents are infinitely-lived, we know from eq. (48) that:

$$1 + g_{K,t} \equiv \frac{K_{t+1}}{K_t} = \left[\frac{K_t}{K_{t-1}}\right]^{\alpha} \left[\frac{A_t}{A_{t-1}}\right]^{1-\alpha} \left[\frac{g_{A,t}}{g_{A,t-1}}\right] \left[\frac{1 + r_{t-1} - \left[\frac{(1+g_{K,t-1})}{(1+g_{A,t-1})}\right]^{\alpha}}{1 + r_t - \left[\frac{(1+g_{K,t})}{(1+g_{A,t})}\right]^{\alpha}}\right].$$

Using that at steady state, $\frac{K_{t+1}}{K_t} = \frac{K_t}{K_{t-1}}$ and $g_{A,t} = g_{A,t-1}$, we obtain:

$$1 + g_{K,t} \equiv \frac{K_{t+1}}{K_t} = \frac{A_{t+1}}{A_t} \left[\frac{1 + r_{t-1} - \left[\frac{(1+g_{K,t-1})}{(1+g_{A,t-1})}\right]^{\alpha}}{1 + r_t - \left[\frac{(1+g_{K,t})}{(1+g_{A,t})}\right]^{\alpha}} \right]^{\frac{1}{1-\alpha}}.$$

Along BGP, l.h.s is constant. R.h.s is constant if

$$1 + r_{t-1} - \left[\frac{(1+g_{K,t-1})}{(1+g_{A,t-1})}\right]^{\alpha} = 1 + r_t - \left[\frac{(1+g_{K,t})}{(1+g_{A,t})}\right]^{\alpha}.$$

This is true if $g_{K,t} = g_{A,t}$, which in turn, implies that:

$$\frac{(1+g_{K,t-1})}{(1+g_{A,t-1})} = \frac{(1+g_{K,t})}{(1+g_{A,t})} = 1,$$

which yields:

$$r_{t-1} = r_t.$$

Thus, the condition for balanced growth path is

$$g_K = g_A. \tag{97}$$

Similarly, it is known from eq. (49) that when patents last for one period,

$$1 + g_{K,t} \equiv \frac{K_{t+1}}{K_t} = \left[\frac{K_t}{K_{t-1}}\right]^{\alpha} \left[\frac{A_t}{A_{t-1}}\right]^{1-\alpha} \left[\frac{g_{A,t}}{g_{A,t-1}}\right]. \tag{98}$$

Using that at steady state, $\frac{K_{t+1}}{K_t} = \frac{K_t}{K_{t-1}}$ and $g_{A,t} = g_{A,t-1}$, we obtain

$$g_K = g_A. \tag{99}$$

Thus, it can be deduced from eqs. (97) and (99) that both the cases of infinitely-lived and one-period patents yield the same steady condition, that is,

$$g_K = g_A.$$

D Derivations which prove that $\lambda + \phi < 1$ is a necessary condition for the steady state under the two technology regimes

We first consider the case when the value of λ or ϕ falls such that $\lambda + \phi < 1$. When patents are one-period lived, we know from eq. (39) that the wage rate for both innovation and imitation economies is given by:

$$w_t = \alpha(1-\alpha)\frac{Y_t}{H_t}g_{A,t},$$

where $g_{A,t} = \dfrac{A_{t+1} - A_t}{A_t}$. Further, we know from eqs. (58) and (57) that the BGP is given by:
Imitation regime:

$$g_K = g_Y = g_A = [(1+g_H)]^{\frac{\lambda}{2-\phi}}(1+g_{\bar{A}})^{\frac{1}{2-\phi}} - 1;$$

Innovation regime:

$$g_K = g_Y = g_A = [(1+g_H)]^{\frac{\lambda}{1-\phi}} - 1.$$

Since $\lambda + \phi < 1$ implies that $\dfrac{\lambda}{1-\phi} < 1$, it can be deduced that $g_Y < g_H$, that is, the aggregate output will grow at a lower rate as compared to aggregate human capital under both the technology regimes. Resultantly, it can be inferred that the wage rate will fall over time for the case of one-period patents.

Similarly, when patents are infinitely-lived, we know from eq. (37) that the wage rate under both the technology regimes can be expressed as:

$$w_t = \frac{\alpha(1-\alpha)}{1 + r_t - \left[\dfrac{(1+g_{K,t})}{(1+g_{A,t})}\right]^{\alpha}}\frac{Y_t}{H_t}g_{A,t},$$

where $g_{K,t} = \dfrac{K_{t+1} - K_t}{K_t}$ and $g_{A,t} = \dfrac{A_{t+1} - A_t}{A_t}$. It is known that $g_Y = g_K = g_A$ as the economy is initially on the BGP. This implies that r_t given by eq. (23) as:

$$r_t = \alpha^2\left[\frac{Y_t}{K_t}\right],$$

is constant. Thus, it can be deduced that wage rate will decline over time when patents are infinitely-lived. Now, from eq. (5), we have:

$$c_{1,t} = \frac{w_t h_t}{1+\beta_1+\beta_2};$$
$$s_t = \frac{\beta_1 w_t h_t}{1+\beta_1+\beta_2}.$$

Thus, it can be inferred that per capita consumption and savings rate in absolute terms fall as the wage rate falls over time under both the technology regimes.

Alternatively, if the the value of λ or ϕ rises such that $\lambda+\phi>1$ along the BGP, then it can be observed from eqs. (58) and (57) that:

$$g_Y > g_H,$$

as $\dfrac{\lambda}{1-\phi} > 1$ when $\lambda+\phi>1$. Thus, similar to the case when $\lambda+\phi<1$, it follows from eqs. (37) and (39) that the wage rate will rise over time for both the cases of infinitely-lived and one-period lived patents. This, in turn, implies that per capita consumption and savings rate in absolute terms rise as the wage rate rises over time.

E Derivations of Eqs. (71) and (72)

We know that $(1+g_H) = (1+g_h).n$. Differentiating both the sides w.r.t θ yields:

$$\frac{\partial g_H}{\partial \theta} = (1+g_h)\frac{\partial n}{\partial \theta} + n\frac{\partial g_h}{\partial \theta}. \tag{100}$$

When $\theta > \frac{\mu}{\tau\epsilon}$, we know from Lemma 1 that:

$$\frac{\partial n_t}{\partial \theta} = -\frac{\mu\beta_2(1-\epsilon)}{(1+\beta_1+\beta_2)(\tau\theta-\mu)^2},$$

and it is given $(1+g_h) = \left[\dfrac{\epsilon(\tau\theta-\mu)}{(1-\epsilon)}\right]^{\epsilon}$ from eq 9. Differentiating g_h w.r.t θ, we get that:

$$\frac{\partial g_h}{\partial \theta} = \left[\frac{\epsilon(\tau\theta-\epsilon)}{1-\epsilon}\right]^{\epsilon}.\frac{\epsilon\tau}{\tau\theta-\mu} = \frac{(1+g_h)\epsilon\tau}{\tau\theta-\mu}. \tag{101}$$

Substituting this into eq. (100), we get that:

$$\frac{\partial g_H}{\partial \theta} = (1+g_H)\frac{-\mu\beta_2(1-\epsilon)}{(1+\beta_1+\beta_2)(\tau\theta-\mu)^2} + (1+g_h)\frac{\epsilon\tau}{\tau\theta-\mu} * n,$$

Substituting for n from eq. (3.8),

$$= (1+g_h)\left[\frac{\epsilon\tau\beta_2\theta(1-\epsilon)}{(1+\beta_1+\beta_2)(\tau\theta-\mu)^2} - \frac{\mu\beta_2(1-\epsilon)}{(1+\beta_1+\beta_2)(\tau\theta-\mu)^2}\right]$$
$$= (1+g_h)[\epsilon\tau\theta-\mu]\frac{\beta_2(1-\epsilon)}{(1+\beta_1+\beta_2)(\tau\theta-\mu)^2}.$$

Alternatively, when $\theta \leq \frac{\mu}{\tau\epsilon}$, we know from Lemma 1 that:

$$\frac{\partial n_t}{\partial\theta} = \frac{\beta_2\epsilon}{(1+\beta_1+\beta_2)\mu},$$

and it is given $(1+g_h) = \mu^{\epsilon}$ from eq. (9). Differentiating g_h w.r.t θ yields:

$$\frac{\partial g_h}{\partial\theta} = 0. \tag{102}$$

Substituting this into eq. (100), we get that:

$$\frac{\partial g_H}{\partial\theta} = (1+g_H)\frac{\beta_2\epsilon}{(1+\beta_1+\beta_2)\mu} = \frac{\beta_2\epsilon}{(1+\beta_1+\beta_2)\mu^{1-\epsilon}}$$

This completes derivation of eqs. (71) and (72).

F Derivations of Eqs. (75), (76), (77) and (78)

The per capita income is $y_t = \frac{Y_t}{L_t}$. The growth rate of per capita income can be expressed as:

$$(1+g_y) = \frac{(1+g_Y)}{n}. \tag{103}$$

Under innovation regime, substituting for $(1+g_Y)$ from eq. (67) and simplifying, we get:

$$(1+g_y) = (1+g_h). \tag{104}$$

Taking log on both sides, we get that:

$$log(1+g_y) = log(1+g_h). \tag{105}$$

Differentiating w.r.t θ yields:

$$\frac{1}{1+g_y}\frac{\partial g_y}{\partial\theta} = \frac{1}{(1+g_h)}\frac{\partial g_h}{\partial\theta}, \tag{106}$$

Substituting for $\dfrac{\partial g_h}{\partial \theta}$ from eq. (101) when $\theta > \frac{\mu}{\tau\epsilon}$, we derive that:

$$\frac{\partial g_y}{\partial \theta} = (1+g_y)\frac{\epsilon\tau}{\tau\theta-\mu}. \tag{107}$$

We, next consider the case where $\theta \leq \frac{\mu}{\tau\epsilon}$.
Substituting for $\dfrac{\partial g_h}{\partial \theta}$ from eq. (102), we derive that:

$$\frac{\partial g_y}{\partial \theta} = 0. \tag{108}$$

Similarly, we derive the expression for $\dfrac{\partial g_y}{\partial \theta}$ under imitation regime.
Substituting for $(1+g_Y)$ from eq. (57) in eq. (103), yields:

$$(1+g_y) = (1+g_{\bar{A}})^{\frac{1}{2-\phi}}(1+g_h)^{\frac{\lambda}{2-\phi}} n^{\frac{\lambda+\phi-2}{2-\phi}}. \tag{109}$$

It is known from Proposition 3.1 that $\lambda+\phi = 1$ along the steady state. Thus, we get:

$$(1+g_y) = (1+g_{\bar{A}})^{\frac{1}{2-\phi}}(1+g_h)^{\frac{\lambda}{2-\phi}} n^{\frac{-1}{2-\phi}}. \tag{110}$$

Taking log on both sides,

$$log(1+g_y) = \frac{1}{(2-\phi)}log(1+g_{\bar{A}}) + \frac{\lambda}{(2-\phi)}log(1+g_h) - \frac{1}{2-\phi}logn, \tag{111}$$

Differentiating w.r.t θ yields:

$$\frac{1}{1+g_y}\frac{\partial g_y}{\partial \theta} = \frac{\lambda}{2-\phi(1+g_h)}\frac{\partial g_h}{\partial \theta} - \frac{1}{2-\phi(n)}\frac{\partial n}{\partial \theta}, \tag{112}$$

When $\theta > \frac{\mu}{\tau\epsilon}$, substituting for n from eq. (8), $\dfrac{\partial n}{\partial \theta}$ from Lemma 1 and $\dfrac{\partial g_h}{\partial \theta}$ from eq. (101), we derive that:

$$\frac{\partial g_y}{\partial \theta} = \frac{(1+g_y)}{2-\phi}\left[\frac{\lambda\epsilon\tau}{\tau\theta-\mu} + \frac{\mu}{\theta(\tau\theta-\mu)}\right]. \tag{113}$$

Similarly, when $\theta \leq \frac{\mu}{\tau\epsilon}$, substituting for n from eq. (8), $\dfrac{\partial g_h}{\partial \theta}$ from eq. (102) and $\dfrac{\partial n}{\partial \theta}$ from Lemma 1 in eq. (112) yields:

$$\frac{\partial g_y}{\partial \theta} = \left[\frac{-(1+g_y)}{\theta(2-\phi)}\right]. \tag{114}$$

This completes derivations of eqs. (75), (76), (77) and (78).

G Derivations of Eqs. (79) and (80)

We know that:

$$(1 + g_H) = (1 + g_h).n \tag{115}$$

Differentiating both the sides w.r.t ϵ, we get that:

$$\frac{\partial g_H}{\partial \epsilon} = (1 + g_h)\frac{\partial n}{\partial \epsilon} + n\frac{\partial g_h}{.}\partial \epsilon \tag{116}$$

When $\theta > \frac{\mu}{\tau\epsilon}$, we know from the interior solution of eq. (9) that:

$$1 + g_h = \left[\frac{\epsilon(\tau\theta - \mu)}{(1 - \epsilon)}\right]^{\epsilon}. \tag{117}$$

Taking log on both sides,

$$log(1 + g_h) = \epsilon log\epsilon + \epsilon log(\tau\theta - \mu) - \epsilon log(1 - \epsilon), \tag{118}$$

Differentiating w.r.t ϵ, we get the following expression:

$$\frac{1}{1 + g_h}\frac{\partial g_h}{\partial \epsilon} = 1 + log\epsilon + log(\tau\theta - \mu) + \frac{\epsilon}{1 - \epsilon} - log(1 - \epsilon), \tag{119}$$

which on simplification reduces to:

$$\frac{\partial g_h}{\partial \epsilon} = (1 + g_h)\left[log\frac{1}{(1 - \epsilon)} + \frac{\epsilon(\tau\theta - \mu)}{1 - \epsilon}\right]. \tag{120}$$

Also, from Lemma 2, we have $\frac{\partial n_t}{\partial \epsilon} = \frac{-\beta_2\theta}{(1 + \beta_1 + \beta_2)(\tau\theta - \mu)}$. Substituting for $\frac{\partial n}{\partial \epsilon}$ from Lemma 2 and $\frac{\partial g_h}{\partial \epsilon}$ from eq. (120) into eq. (116), we derive that:

$$\frac{\partial g_H}{\partial \epsilon} = \frac{-\beta_2\theta(1 + g_h)}{(1 + \beta_1 + \beta_2)(\tau\theta - \mu)} + (1 + g_h)n\left[\frac{1}{(1 - \epsilon)} + log\frac{\epsilon(\tau\theta - \mu)}{1 - \epsilon}\right] \tag{121}$$

Substituting for $\frac{\beta_2\theta}{(1 + \beta_1 + \beta_2)(\tau\theta - \mu)}$ from eq. (8), yields:

$$= (1 + g_h)\left[n\left[\frac{1}{(1 - \epsilon)} + log\frac{\epsilon(\tau\theta - \mu)}{1 - \epsilon}\right] - \frac{n}{1 - \epsilon}\right] \tag{122}$$

$$= (1 + g_H)log\left[\frac{\epsilon(\tau\theta - \mu)}{1 - \epsilon}\right]. \tag{123}$$

We next derive the expression for $\frac{\partial g_H}{\partial \epsilon}$ when $\theta \leq \frac{\mu}{\tau\epsilon}$.
When $\theta \leq \frac{\mu}{\tau\epsilon}$, it is known from eq. (9) that:

$$(1+g_h) = \mu^{\epsilon}. \tag{124}$$

Taking log on both sides,

$$log(1+g_h) = \epsilon log\mu, \tag{125}$$

Differentiating g_h w.r.t ϵ yields:

$$\frac{\partial g_h}{\partial \epsilon} = (1+g_h)log\mu. \tag{126}$$

Further, we derive know from Lemma 2 that $\frac{\partial n_t}{\partial \epsilon} = \frac{\beta_2\theta}{(1+\beta_1+\beta_2)\mu}$. Substituting for $\frac{\partial n}{\partial \epsilon}$ and $\frac{\partial g_h}{\partial \epsilon}$ from eq. (126) into eq. (116), we deduce that:

$$\frac{\partial g_H}{\partial \epsilon} = \frac{\beta_2\theta(1+g_h)}{(1+\beta_1+\beta_2)\mu} + (1+g_h)nlog\mu. \tag{127}$$

Substituting for $\frac{\beta_2\theta}{(1+\beta_1+\beta_2)\mu)}$ from eq. (8) when $\theta \leq \frac{\mu}{\tau\epsilon}$, we derive that:

$$= (1+g_H)\left[\frac{1}{\epsilon} + log\mu\right]. \tag{128}$$

This completes the derivations of eqs. (79) and (80).

H Derivations of Eqs. (81), (82), (83) and (84)

We first, consider the innovation regime when $\theta > \frac{\mu}{\tau\epsilon}$. It is known from eq. (104) that the growth rate of per capita income is given by:

$$(1+g_y) = (1+g_h). \tag{129}$$

Taking log on both sides, we get,

$$log(1+g_y) = log(1+g_h). \tag{130}$$

Differentiating eq. (130) w.r.t ϵ, we have,

$$\frac{1}{1+g_y}\frac{\partial g_y}{\partial \epsilon} = \frac{1}{(1+g_h)}\frac{\partial g_h}{\partial \epsilon}, \tag{131}$$

Substituting for $\frac{1}{1+g_h}\frac{\partial g_h}{\partial \epsilon}$ from eq. (120), we get that:

$$\frac{\partial g_y}{\partial \epsilon} = (1+g_y)\left[\frac{1}{1-\epsilon} + log\frac{\epsilon(\tau\theta-\mu)}{(1-\epsilon)}\right]. \tag{132}$$

We next, consider the case where $\theta \leq \frac{\mu}{\tau\epsilon}$.

Substituting for $\frac{1}{1+g_h}\frac{\partial g_h}{\partial \epsilon}$ from eq. (126) in eq. (131), we get:

$$\frac{\partial g_y}{\partial \epsilon} = log\mu(1+g_y) \tag{133}$$

Similarly, we derive the expression for $\frac{\partial g_y}{\partial \epsilon}$ under imitation regime.

We know from eq. (111) that:

$$log(1+g_y) = \frac{1}{(2-\phi)}log(1+g_{\bar{A}}) + \frac{\lambda}{(2-\phi)}log(1+g_h) - \frac{1}{2-\phi}logn \tag{134}$$

Differentiating w.r.t ϵ yields:

$$\frac{1}{1+g_y}\frac{\partial g_y}{\partial \epsilon} = \frac{\lambda}{2-\phi(1+g_h)}\frac{\partial g_h}{\partial \epsilon} - \frac{1}{2-\phi(n)}\frac{\partial n}{\partial \epsilon}. \tag{135}$$

When $\theta > \frac{\mu}{\tau\epsilon}$, substituting for n from eq. (8), $\frac{\partial n}{\partial \epsilon}$ from Lemma 2 and $\frac{\partial g_h}{\partial \epsilon}$ from eq. (120), we derive that:

$$\frac{\partial g_y}{\partial \epsilon} = \frac{\lambda(1+g_y)}{2-\phi}\left[\frac{1}{1-\epsilon} + log\frac{\epsilon(\tau\theta-\mu)}{(1-\epsilon}\right] + \frac{(1+g_y)}{(2-\phi)(1-\epsilon)}. \tag{136}$$

Alternatively, when $\theta \leq \frac{\mu}{\tau\epsilon}$, substituting for n from eq. (8), $\frac{\partial g_h}{\partial \epsilon}$ from eq. (126) and $\frac{\partial n}{\partial \epsilon}$ from Lemma 2 in eq. (135) yields:

$$\frac{\partial g_y}{\partial \epsilon} = \frac{\lambda(1+g_y)log\mu}{\epsilon(2-\phi)} - \frac{(1+g_y)}{\epsilon(2-\phi)}. \tag{137}$$

This completes the derivations of eqs. (81), (82), (83) and (84).

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