Discussion Papers in Economics

Imperfect Certification under Cournot Duopoly

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Discussion Paper 15-03

Centre for International Trade and Development

School of International Studies

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India

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Abstract

Environmental quality is often a credence good and consumers are unable to distinguish between green and brown products. The paper aims to investigate the role of certification in providing information about product quality and reducing market inefficiencies when the certification process is imperfect. We consider a duopoly in a vertically differentiated product model where firms compete in quantities. The papers shows that in the absence of labelling, the brown firm drives out the green firm if the cost of producing green product is sufficiently high. If both firms produce positive quantities in the market, the green firm covers a higher market share and obtains larger revenue. We then characterise pooling and separating equilibrium under imperfect certification contingent on certification fee. The paper shows that under imperfect certification, it is not optimal to subsidize certification.

Keywords: Eco-Labelling; Environmental quality; Imperfect Certification; Incomplete Information; Vertical differentiation.

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1 INTRODUCTION

Consumers' concern for the environment has grown in recent years. This concern is getting translated into their willingness to pay for environmental attributes such as sustainable, recycled, non-toxic and biodegradable products. There is ample empirical evidence to show that firms take note of consumers willingness to pay a premium for environmentally friendly products and differentiate their products on the basis of environmental attributes. A problem with environmental attributes is that they are often not observable. Although consumers prefer environment friendly products, they are unable to ascertain the quality attributes of the products either on inspection or even after consumption. Such goods whose quality is not verifiable even after consumption are called credence goods (Nelson, 1970). Examples of such goods include organically produced food, green electricity, energy efficiency, eco-friendly soaps, laundry detergents, toilet paper, etc.

The credence goods nature of these products give rise to inefficiency in the market outcome. Since these attributes are not observable, producers of inferior quality have incentives to cheat. Consumers anticipate this and are willing to pay a lower amount for the products available in the market. This may lead to a breakdown of the market for environmentally superior firms or in other words, there would be adverse selection (Akerlof, 1970). One possible solution for the informational asymmetry problem is to signal the attributes by using eco-labels.

Eco-labelling informs consumers about the quality attribute by informing that the product complies with prespecified standards. A wide variety of eco-labels have been introduced in various countries, for example, German Blue Angel (1977), Nordic Swan (1989), Japanese Eco-Mark (1989), Swedish Environmental Choice (1990), U.S. Energy Star (1992), EU Eco-label (1992), Thai Green Label (1994), and Dolphin Safe Tuna. The empirical studies have found that consumers are willing to pay more for green products. Bjorner et al. (2003) conducted an empirical analysis for the Nordic Swan Label using data for Danish consumers from 1997 - 2001. It reported statistically significant levels of consumer's preferences for more expensive labelled toilet paper brands and laundry detergents. Shen (2008) found that Chinese consumers prefer environmental conservation more than life convenience, and are willing to pay 8.71 - 9.51 percent for products awarded with China eco-label. Marette (2012) showed that the additional information on organic products about its pesticides use significantly increased the consumers' willingness to pay. Teisl et al. (2002) provided evidence that consumers responded to the implementation of dolphin-safe labelling. Griffith and Nesheim (2008) used hedonic prices to show 80 percent of the households surveyed were willing to pay a higher price for organic food products in U.K.

Eco-labels may differ with respect to content, precision and reliability. The certification process entails cost and thus may not be perfect. In addition, firms may have incentives to mislabel. The paper aims to investigate the role of certification in providing information and reducing market inefficiencies when the "*certification process is imperfect*". Much of the existing literature analysing eco-labels assumes perfect certification. The paper intends to contribute to the literature by investigating the equilibria that may occur with eco-labelling under imperfect certification. Our framework models certification as a noisy test under vertical product differentiation. It allows for the possibility of errors in the third party certification by considering random monitoring.

An emerging literature in environment economics has modelled consumer's preference to purchase from green firms using vertically differentiated product models. A large part of this literature has investigated market equilibrium and government regulation assuming that consumers are able to observe the environmental attribute of the products (Arora and Gangopadhyay 1995; Bansal 2008; Amacher et al. 2004 are some of the studies). Ibeas (2007) showed that if the marginal cost of producing environmentally differentiated products is sufficiently low, then an increase in the proportion of green consumers results in reduction of pollution. However, when the proportion of green consumers is low, then both firms don't care about the environment attributes. Thus, there is less product differentiation and cost advantage allows brown firm to capture the market.

The theoretical literature has also modelled situations where only producers know the quality of the product and consumers do not observe it. It then discusses how eco-labelling can remove market inefficiencies. (See Marette et al. (1999); Zago and Pick (2004); Auriol and Schilizzi (2013); Arguedas and Blanco (2013), etc.) This literature, however, assumed perfect certification. Marette et. al. (1999) analyzed a three stage game under imperfect competition, using cartel theory. The firms decide whether to certify their goods independently or to form a cartel to share the labelling cost and/or collude in quantities. The symmetric Nash equilibrium of the game is a separating equilibrium in which if the certification cost is high compared to the profit, the high quality firms can share the cost of certification, and enforce quantity collusion. Analyzing perfect certification under horizontal product differentiation, Das (2013) shows that the certifier charges a very high fees if the degree of horizontal product differentiation is below a certain threshold. Then only one firm opts for certification and produces the highest quality, the other firm does not opt for certification and produce low quality products. However, above the threshold value of the degree of product differentiation, there is less competition among the firms and thus, both firms opt for certification and produce high quality.

There have been attempts in the literature to model mislabeling by firms. That is, despite the certifying agency accurately certifying the products, the firms cheat by attaching spurious labels to their products (De and Nabar (1991); Giannakas (2002); Liang and Jensen (2007); Baksi and Bose (2007)). Giannakas (2002) examines the consequences of mislabelling on consumers welfare in organic food market. In an empirical study on food certification, Ependitis (1998) reports that about 15 -40 percent of the organic labelled products are mislabelled. Mislabelling creates uncertainty about the nature of the organic labelled product and drives out some consumers from the market. Liang and Jensen (2007) provide a theoretical framework for analyzing mislabelling resulting in imperfect food certification. While Marette et al. (1999) characterize separating equilibrium under perfect certification, Liang and Jensen (2007) characterise pooling equilibrium under imperfect certification. Discussing the role of monitoring and enforcement process, the later paper showed that the fraudulent output can be eliminated with high monitoring rate and the imposition of penalty on being caught. Hamilton and Zilberman (2006) assumed that technology used is not observed by the consumers due to which there is a possibility of fraudulent brown products to be sold in the green markets. The paper showed that if there are entry barriers, the extent of fraud in the green market is reduced. Eco-certification policies increase the cost for use of green techniques, thereby reducing fraud. However, the extent of improvement in social welfare depends on the monitoring and enforcement effort.

Since testing and certification entail cost, the certifying agency may test only a sample of the products and there is a possibility of an error in the accuracy of the certification process. There has been very little theoretical work modelling such imperfect certification. Mason (2011), and Botttega and Freitas (2013) allow for the possibility of errors by the certification agency. In a perfectly competitive market set up, Mason (2011) analyses imperfect certification where green firms are more likely to pass the certification test than brown firms. It identifies the conditions for the existence of separating, pooling and partial pooling equilibria. The separating equilibrium can exist when the test cost is sufficiently large. Under separating equilibrium, green firms produce a larger amount and brown firms produce a smaller amount as compared to no information equilibrium, and this has a welfare improving effect. However, the ultimate impact on welfare is not clear as the profits of green and brown firms are lower in a separating equilibrium than in no information case. The pooling equilibrium, on the other hand, exists when the test cost is sufficiently small. As compared to the no information case, in a pooling equilibrium while the green firms could be either better off or worse off, the brown firms are at least as well off. Bottega and Freitas (2013) analyse imperfect certification under a vertical product differentiation model with price competition assuming markets to be fully covered. The paper shows that the equilibrium price equals marginal costs if either both firms are awarded label or none of them is awarded label. However, if one firm is awarded label, then firms are able to differentiate. The paper shows separating equilibrium is Perfect Bayesian equilibrium and the label allows firms to extract benefits from differentiation.

The present study aims to contribute to the literature examining role of certification in information provision. In a vertically differentiated product framework, certification is modelled as a noisy test. In contrast to Bottega and Freitas (2013), we model quantity competition and allow the market to be partially covered.

The paper has wider applications and is applicable to labelling where the attribute under consideration is social in nature. For example, there is an awareness against products manufactured using child labor. Consumers cannot observe whether the manufacturing process has used child labor or not and rely on the certification provided by the producer.

The paper is structured as follows. Section 2 and 3 develops the duopoly model under vertical product differentiation framework. In these section, we first discuss the benchmark case of complete information, followed by incomplete information. Under incomplete information the paper models two situations, one in which there is absence of labelling, and the other in which third party certifies the product, and there is imperfect certification. Finally Section 4 contains the concluding remarks.

2 MODEL STRUCTURE

Consider a vertical differentiation model with two firms producing products with different technologies. The "Green Firm (g)" produces goods with non-polluting technology and the "Brown Firm (b)" produces goods with polluting technology. Let s_g and s_b denote the qualities of the green and brown good, respectively. Without loss of generality, assume $s_g > s_b$.

A.1: Let the green firm cost function be denoted by $C(s_g)$ and brown firm cost function by $C(s_b)$. Assume fixed costs of providing quality is given by -

$$C(s_i) = ks_i, i = g, b$$

where k is positive parameter. As $s_g > s_b$, the green production is more expensive as compared to brown production.

There exists a continuum of consumers who are identical in all respects except their preferences for the quality attribute. The utility function of consumer is given by -

$$U(\theta, p_i) = \theta E(s) + y - p_i \tag{1}$$

where θ is the marginal willingness to pay for environmental quality of the product, $E(s)$ is the quality of the product available in the market as perceived by the consumer, y is the income of the consumer and p is the price charged by firm i, where $i = g, b$. This utility function implies that all consumers unanimously prefer a higher quality at a given price and that consumers with higher θ have a higher willingness to pay for higher quality.

A.2: Assume the parameter θ is uniformly distributed over $[0, 1]$ and the distribution of θ is denoted by $F(\theta)$.

As consumers do not observe the true quality of the product, their consumption decisions are based on the perceived quality. Assume qualities to be exogenously given. Firms compete in quantities and we allow for the market to be partially covered. As a benchmark case, we first discuss the situation of complete information. Next, we analyse the incomplete information situation under absence of eco-labelling and the role played by third party certification.

2.1 Complete Information

Under complete information, consumers observe the quality level chosen by the firms. Substituting $E(s) = s_i$ in (1), we get the utility function of the consumer as -

$$U(\theta, s_i) = \theta s_i + y - p_i \tag{2}$$

The two qualities corresponds to the two types of firms, $i =$ Green (g) and Brown (b). The consumer who is indifferent between purchasing a green good and brown good is given by $\theta_1 = \frac{p_g - p_b}{s_g - s_b}$. The consumer who is indifferent between buying and not buying is given by

$\theta_2 = \frac{p_b}{s_b}$. The demand for the green good and the brown good is $q_g = 1 - \theta_1$ and $q_b = \theta_1 - \theta_2$, respectively. Figure 1 shows the covered and uncovered market shares. The consumers with willingness to pay less than θ_2 don't buy the product, between θ_2 and θ_1 buy from brown firm, and with willingness to pay greater than θ_1 buy from the green firm.

The firms maximise profits to choose quantities. The profit of the firm i is given by -

$$\pi_i = p_i q_i - k s_i$$

We rearrange the prices in terms of quantities of two firms and subsititute in the profit equation. The Nash equilibrium of the game given by maximising firms profits with respect to quantity is -

$$q_g^* = \frac{2s_g - s_b}{4s_g - s_b} \quad (3)$$

$$q_b^* = \frac{s_g}{4s_g - s_b} \quad (4)$$

where a superscript * denotes value of the variable in equilibrium.

The second order conditions for profit maximisation are satisfied. The prices of the green and brown firm are given by -

$$p_g^* = \frac{2s_g^2 - s_b s_g}{4s_g - s_b} \quad (5)$$

$$p_b^* = \frac{s_b s_g}{4s_g - s_b} \quad (6)$$

The profits of the green and brown firm are given by -

$$\pi_g = \frac{s_g(2s_g - s_b)^2}{(4s_g - s_b)^2} - k s_g \quad (7)$$

$$\pi_b = \frac{s_g^2 s_b}{(4s_g - s_b)^2} - k s_b \quad (8)$$

In our model, we allow for the market to be uncovered. The necessary condition for an equilibrium is $0 \leq \theta_2 \leq \theta_1 \leq 1$. The uncovered market share is given by $\theta_2 = \frac{s_g}{4s_g - s_b} > 0$. Also, as $\theta_1 = \frac{2s_g}{4s_g - s_b}$, the above condition is satisfied for the model.

Proposition 1: Assume $s_g, s_b > 0$ and $k < min(\frac{(2s_g - s_b)^2}{(4s_g - s_b)^2}, \frac{s_g^2}{(4s_g - s_b)^2})$, in the market equilibrium under complete information,

(a) Both firms produce in the market, i.e., $q_g^* > 0, q_b^* > 0$.

(b) The price charged and the quantity produced by the green firm is greater than the price charged and quantity produced by the brown firm, respectively.

(c) The revenue of each firm is increasing in its own quality, given the quality of the other firm and decreasing in the quality of the other firm, given its own quality.

Proof: See Appendix A.1.

The results stated in Proposition 1 show that the green firm's revenue is increasing in product differentiation, but opposite is the case for the brown firm. The result is different from the case where firms compete in prices, as there both firms' profits are increasing in product differentiation, and if firms were allowed to choose qualities maximal differentiation would follow.

2.2 Incomplete Information

Suppose that the consumers cannot observe the good's type. We first discuss market equilibrium in the absence of eco-labelling.

2.2.1 Absence of Eco-Labelling

Suppose firms do not label their product. The consumer can formulate different subjective probabilities (beliefs) concerning which good has higher quality in terms of unobservable characteristics.

For this section, we assume that consumers are able to observe the quantities being produced by the green and the brown firms. Let q_g and q_b be quantities of green and brown firm products available in the market, respectively. As consumers cannot distinguish between the goods, the perceived quality is same for all goods. In other words, there is only one perceived quality. For this section following Mason (2011), probability that a good is green is given by $Pr(g) = \frac{q_g}{q_g+q_b}$ and probability that a good is brown is given by $Pr(b) = \frac{q_b}{q_g+q_b}$. The perceived quality available in the market is given by the weighted average of two qualities, where the weight for each quality is given by the ratio of quantity produced by that quality and total quantity available in the market -

$$E(s) = \frac{q_g}{q_g + q_b} s_g + \frac{q_b}{q_g + q_b} s_b \tag{9}$$

Using (1), the consumer who is indifferent between buying a good or not buying is given by $\overline{\theta}$, where $\overline{\theta} = \frac{p}{E(s)}$. The prices for the green and brown goods are the same. The demand for good is $q = q_g + q_b = 1 - \overline{\theta} = 1 - \frac{p}{E(s)}$. Thus, using equation (9) the price of the good in terms of quantities is given by

$$p = (1 - q_g - q_b)(\frac{q_g s_g + q_b s_b}{q_g + q_b}) \tag{10}$$

For solving the game, each firm chooses quantity to maximise profits. The profit of firm i is given by -

$$\pi_i = pq_i - ks_i \tag{11}$$

Substituting the value of p from equation (10), the profits of the green and brown firm are given as -

$$\pi_g = (1 - q_g - q_b)(\frac{q_g s_g + q_b s_b}{q_g + q_b})q_g - ks_g \tag{12}$$

$$\pi_b = (1 - q_g - q_b)(\frac{q_g s_g + q_b s_b}{q_g + q_b})q_b - ks_b \tag{13}$$

Maximising profits with respect to quantity, gives the best response function of the Green and Brown Firm -

$$\frac{\partial \pi_g}{\partial q_g} = \frac{(q_g + q_b)((q_g - q_g^2 - q_g q_b)s_g + (q_g s_g + q_b s_b)(1 - 2q_g - q_b)) - (q_g s_g + q_b s_b)(q_g - q_g^2 - q_g q_b)}{(q_g + q_b)^2} = 0$$

$$\frac{\partial \pi_b}{\partial q_b} = \frac{(q_g + q_b)((q_b - q_b^2 - q_g q_b)s_b + (q_g s_g + q_b s_b)(1 - 2q_b - q_g)) - (q_g s_g + q_b s_b)(q_b - q_b^2 - q_g q_b)}{(q_g + q_b)^2} = 0$$

Solving the two best response functions simultaneously gives us an equilibrium. Under the constraint $q_g \geq 0$ and $q_b \geq 0$, the unique Nash Equilibrium of the game is -

$$q_g = \frac{2}{3} + \frac{-(4s_g - 2s_b) + 2(s_g^2 + s_b^2 - s_g s_b)^{\frac{1}{2}}}{9(s_g - s_b)} \tag{14}$$

$$q_b = \frac{4s_g - 2s_b - 2(s_g^2 + s_b^2 - s_g s_b)^{\frac{1}{2}}}{9(s_g - s_b)} \tag{15}$$

The calculations of the solution are shown in Appendix B. Also, the second order conditions are satisfied for both the firms (shown in Appendix C).

It might seem intuitive that both firms should produce the same quantity as consumer's cannot distinguish between them and costs are fixed costs. However, in equilibrium $q_g > q_b$. This is because consumers' perceived quality $E(s)$ and thereby price is increasing in q_g. Thus, the willingness to pay also increases with an increase in q_g. For the brown firm, there is a trade off as its profits are increasing in its quantity sold as well as price. These forces work in such a manner that in equilibrium $q_g > q_b$.

As consumers are unable to observe the environmental attributes of the products, the brown firms have an incentive to cheat. The brown firms can pretend to be green firms in order to increase their profits. In the absence of credible mechanisms for information disclosure, the consumers are not able to verify the greenness of the products. For certain parameter values, the green firms are driven out of the market, i.e., there is adverse selection. The proposition below gives the range on the cost parameter for green and brown firm to operate in the market.

Proposition 2: For different values of k, the equilibrium configurations are the following

(a) Both firms produce in the market earning positive profits for $0 \leq k \leq \frac{2(s_g+s_b+A)}{27s_g} - \frac{B}{s_g}$,

(b) Only brown firm produces in the market for $\frac{2(s_g+s_b+A)}{27s_g} - \frac{B}{s_g} < k \leq \frac{B}{s_b}$; this corresponds to adverse selection, where polluter (brown firm) drives the environmental friendly (green firm) out of the market,

(c) None of the firms operate in the market for $k > \frac{B}{s_b}$,

where $A \equiv (s_g^2 + s_b^2 - s_g s_b)^{\frac{1}{2}}$ and $B \equiv \frac{2(s_g+s_b+A)(2s_g-s_b-A)}{81(s_g-s_b)}$.

Proof: See Appendix A.2

Parts (a) and (c) are straightforward; if k is very small, both firms can survive in the market, however, if k is extremely large, none of the firms can make positive profit. Our interest lies in part (b), which shows adverse selection. If k is not very small, for sufficiently large gap between the two qualities, there may be situations of adverse selection. Therefore, polluter (brown firm) drives out of the market the environmentally friendly firm (Green).

3 THIRD PARTY CERTIFICATION

Third party certification allows firms to signal their product quality and differentiate their products. Suppose that the third party conducts a certification test for the firm's product at a specified cost, C. The certification provides consumers with credible information on the quality attributes of the product. The certification agencies adopt random monitoring, as it is costly to monitor continuously (Mason, 2011). Thus, there is possibility of error in the certification process. The third party could mistakenly certify some brown firm products or may not certify certain green firm products. Thus, using equation (1), we now define the utility function of the consumer as -

$$U(\theta, s) = \theta E(s|j) + y - p_j \tag{16}$$

where $j = c$ (certified) or un (uncertified), $E(s|j)$ is the perceived quality of the product, which is conditional on whether the product is being certified or uncertified, y is income of the consumer and p_j is the price of the good.

Consider the following extensive form of the game with two firms - Green and Brown firm, certifying agency and consumers (shown in Figure 2).

- Each firm knows its type, but firms' type is unobservable by the consumers and the certifying agency. For the purpose of tractability, now onwards, we assume that ex-ante consumers perceive that a firm could be green or brown with an equal probability, i.e., probability of each firm type being chosen is $\frac{1}{2}$.

- Conditional on their types, the firms simultaneously choose whether to seek certification test or not seek certification test. Let σ_i be the probability of firm i seeking certification ($i = g$ or b) and $1-\sigma_i$ be the probability of firm i not seeking certification, where $\sigma_i \; \epsilon \; \{0, 1\}$.

- If a firm seeks certification, the certifying agency adopts random monitoring to label the firm's product. The firms which seek certification and pass the test are labelled as "Certified (c)". The firms which either don't seek certification test or fail are labelled as "Uncertified (un)". The probability of green firm passing the test is ϕ_g and the brown firm passing the test ϕ_b. It is reasonable to assume $0 \leq \phi_b < \phi_g \leq 1$.

- Next, certified and uncertified green and brown firms simultaneously choose quantities to maximise their profits.

- Finally, consumers observe the label on the product, formulate beliefs about the firm's type and accordingly purchase the quantity of the products. The consumer's beliefs are based on the firm decision to seek the certification and the probability of firm passing the test.

A pure strategy for the green firm is $\psi_g = \{\sigma_g, q_g\}$, and the pure strategy for brown firm is $\psi_b = \{\sigma_b, q_b\}$. The certifying agency gets a chance to take an action once firms have taken its decision on seeking or not seeking certification. Let the decision for the certifying agency $\Omega \in \{c, un\}$. The certifying agency labels the firms product passing the test, and consumers formulates the belief about the quality of the product. Since a priori consumers believe that the product could be coming from green or brown firm with probability 1/2, prior belief on expected quality is $E(s) = \frac{s_g + s_b}{2}$. However, when consumers face labelled products, the beliefs are updated using Bayes' rule. Let $\beta(s_g|c)$ denote consumer's updated belief that the good is of high quality provided it is certified and $\beta(s_g|un)$ denote consumer's updated belief that the good is of high quality provided it is not certified, where β represents probability. Thus, $\beta(s_b|c) = 1 - \beta(s_g|c)$ denote consumers' updated belief that the good is of low quality provided it is certified; the probability $\beta(s_b|un) = 1 - \beta(s_g|un)$ denote consumers' updated belief that the good is of low quality provided it is not certified. The expected quality of the certified unit is $E(s|c)$ and expected quality of the uncertified good is $E(s|un)$, where they are defined as

$$E(s|c) = \beta(s_g|c)s_g + \beta(s_b|c)s_b \quad (17)$$

$$E(s|un) = \beta(s_g|un)s_g + \beta(s_b|un)s_b \quad (18)$$

Assume that the probability of firm seeking certification and probability of firm passing the test are independent and exogenously given. The cases where either both the products are certified or both are not certified correspond to the case of no information (discussed above).

We therefore focus on the case where only one firm product is certified. Using Bayes' rule, the updated consumer's belief that the product is of high quality (green), given it is certified is defined as

$$\beta(s_g|c) = \frac{\phi_g\sigma_g(1-\sigma_b) + \phi_g(1-\phi_b)\sigma_g\sigma_b}{\phi_g\sigma_g(1-\sigma_b) + \phi_b\sigma_b(1-\sigma_g) + \phi_g(1-\phi_b)\sigma_g\sigma_b + \phi_b(1-\phi_g)\sigma_g\sigma_b} \tag{19}$$

The numerator is the joint probability that the product is of high quality and is certified. It is obtained as the product of the marginal probability that the product is high quality, i.e., green firm and the conditional probability that product is certified, given it is of high quality. The terms in the numerator of equation (19) indicates that green firm pass the test and is certified, however, the brown firm either doesn't seek certification (First Term) or it seeks certification and fails (Second term), and thus is unlabelled. The denominator is the marginal probability that firm's product is certified. The first and the third term indicate that green firm is certified, and brown firm either doesn't seek the certification or fails. The second and fourth term indicate that brown firm is certified and green firm either doesn't seek certification or fails.

Similarly, the rest of the consumer's beliefs are defined as -

$$\beta(s_g|un) = \frac{\phi_b\sigma_b(1-\sigma_g) + \phi_b(1-\phi_g)\sigma_b\sigma_g}{\phi_b\sigma_b(1-\sigma_g) + \phi_g\sigma_g(1-\sigma_b) + \phi_b(1-\phi_g)\sigma_b\sigma_g + \phi_g(1-\phi_b)\sigma_b\sigma_g} \tag{20}$$

$$\beta(s_b|c) = 1 - \beta(s_g|c) \tag{21}$$

$$\beta(s_b|un) = 1 - \beta(s_g|un) \tag{22}$$

The paper aims to determine the Perfect Bayesian Equilibrium of the game. It includes firm's equilibrium decision on certification tests and consumer's equilibrium beliefs on the product qualities. The consumer's beliefs are consistent with the firm's strategies and derived using Bayes' rule. The firm's strategies are sequentially rational, given the consumer's beliefs.

Definition 1: Third Party Certification Game Perfect Bayesian Equilibrium - The assessment $(\psi_g, \psi_b, \Omega, \beta(.))$ forms a Perfect Bayesian Equilibrium of the third party certification signalling game as -

(a) Given certification agency decision Ω, firms simultaneously choose quantity to maximise their profits.

(b) The consumers' belief satisfy the Bayes' Rule, that is

  (i) $\beta(s_g|c) \in [0,1]$, $\beta(s_g|un) \in [0,1]$ for certification agency certifying the product.

  (ii) If $\sigma_g = 1$ and $\sigma_b = 0$, then $\beta(s_g|c) = 1$ and $\beta(s_g|un) = 0$.

  (iii) If $\sigma_g = \sigma_b = 1$, then $\beta(s_g|c) = \frac{\phi_g(1-\phi_b)}{\phi_g(1-\phi_b)+\phi_b(1-\phi_g)}$ and $\beta(s_g|un) = \frac{\phi_b(1-\phi_g)}{\phi_g(1-\phi_b)+\phi_b(1-\phi_g)}$.

(c) For firms choice ψ_g, ψ_b, the consumer purchases the product which maxmises their utility, and the certification agency certifies the firm's product which pass the test.

Condition (a) and (c) ensure that assessment is sequentially rational. Each firm will choose the quantity that maximises its profits, given the strategy followed by the other firm and the belief structure of the consumer. Condition (b) ensures that consumer's beliefs satisfy Bayes' rule. The consumer's beliefs about the firms product being green or brown are consistent with equilibrium strategies both on and off the equilibrium path.

Analysing the Game

In the first stage of the game, the firms decide whether to go for testing of the product or not. The third party certifier awards label to the firm passing the test. Assume that certification test could involve error, i.e., it is noisy. In the second stage of the game, the firms compete by choosing quantities for their products to maximise profits. The game is solved using backward induction.

Quantity Competition

The firms simultaneously choose quantities to maximise the profits. The consumer who is indifferent between buying a certified and uncertified good is given by $\widetilde{\theta_1} = (p_c - p_{un})/(E(s|c) -$

$E(s|un)$). The consumer who is indifferent between buying an uncertified good and not buying is given by $\widetilde{\theta}_2 = \frac{p_{un}}{E(s|un)}$. The firms which are certified receive price p_c and firms which are uncertified receive price p_{un}. The demand for the certified good and the uncertified good is given by $q_c = 1 - \widetilde{\theta}_1$ and $q_{un} = \widetilde{\theta}_1 - \widetilde{\theta}_2$, respectively.

As the firms fixed costs don't influence the quantity to be produced and there is no variable cost in the model. For solving the game, we maximise the revenue of the certified and uncertified firm ($j = c$ or un), given by -

$$R_j = p_j q_j$$

We rearrange for the prices in terms of the quantities of the two firms and substitute in the revenue equation. The Nash Equilibrium of the game is -

$$q_c^* = \frac{2E(s|c) - E(s|un)}{4E(s|c) - E(s|un)} \tag{23}$$

$$q_{un}^* = \frac{E(s|c)}{4E(s|c) - E(s|un)} \tag{24}$$

The second order conditions for profit maximisation are satisfied (shown in Appendix D). The prices of the certified and uncertified firm are given by -

$$p_c^* = \frac{(2E(s|c) - E(s|un))E(s|c)}{4E(s|c) - E(s|un)} \tag{25}$$

$$p_{un}^* = \frac{E(s|c)E(s|un)}{4E(s|c) - E(s|un)} \tag{26}$$

The revenue of the certified and uncertified firm are given by -

$$R_c = \frac{(2E(s|c) - E(s|un))^2 E(s|c)}{(4E(s|c) - E(s|un))^2} \tag{27}$$

$$R_{un} = \frac{(E(s|c))^2 E(s|un)}{(4E(s|c) - E(s|un))^2} \tag{28}$$

Next, solve for the first stage in which firms decide whether to certify their products or not, based on the beliefs formed by the consumers about the product quality.

Testing Decisions

The producer knows the quality of the product produced; however, the consumer is unable to observe the quality due to credence nature of the product. Thus, consumer forms the beliefs regarding the product quality being sold in the market. The firm's testing decisions depend on the consumer's belief about the product quality. Under this stage, the profits of the green and brown firm are calculated, based on firm's testing decisions. For simplification in notation, we define firms decision to seek certification test as "t", i.e., when $\sigma = 1$, and not seeking certification test as "n" i.e., when $\sigma = 0$. Table 1 gives the payoff matrix for green and brown firm, under the two strategies (t or n).

Table 1: **Payoff Matrix**

Green/Brown Firm	Test (t)	Not Test (n)
Test (t)	$\pi_g(t,t), \pi_b(t,t)$	$\pi_g(t,n), \pi_b(t,n)$
Not Test (n)	$\pi_g(n,t), \pi_b(n,t)$	$\pi_g(n,n), \pi_b(n,n)$

Given the firm strategy and probability of passing the test, the profits for each firm is calculated, using profits calculated in stage 2.

Green Firm -

$$\pi_g(t,t) = \frac{\phi_b(1-\phi_g)(E(s|c))^2 E(s|un) + \phi_g(1-\phi_b)(2E(s|c) - E(s|un))^2 E(s|c)}{(4E(s|c) - E(s|un))^2} - ks_g - C$$

$$\pi_g(t,n) = \frac{\phi_g(2E(s|c) - E(s|un))^2 E(s|c)}{(4E(s|c) - E(s|un))^2} - ks_g - C$$

$$\pi_g(n,t) = \frac{\phi_b(E(s|c))^2 E(s|un)}{(4E(s|c) - E(s|un))^2} - ks_g$$

$$\pi_g(n,n) = \frac{\phi_b(E(s|c))^2 E(s|un)}{(4E(s|c) - E(s|un))^2} - ks_g$$

Brown Firm -

$$\pi_b(t,t) = \frac{\phi_b(1-\phi_g)(2E(s|c)-E(s|un))^2E(s|c)+\phi_g(1-\phi_b)(E(s|c))^2E(s|un)}{(4E(s|c)-E(s|un))^2} - ks_b - C$$
$$\pi_b(t,n) = \frac{\phi_g(E(s|c))^2E(s|un)}{(4E(s|c)-E(s|un))^2} - ks_b$$
$$\pi_b(n,t) = \frac{\phi_b(2E(s|c)-E(s|un))^2E(s|c)}{(4E(s|c)-E(s|un))^2} - ks_b - C$$
$$\pi_b(n,n) = \frac{(E(s|c))^2E(s|un)}{(4E(s|c)-E(s|un))^2} - ks_b$$

The paper focuses on the conditions required for different equilibrium (separating and pooling) to exist. An equilibrium is a separating equilibrium if the two firms choose different testing decisions. In this way, firms separate themselves from one another. On this basis, the consumer is able to identify the green and brown firm, when making their purchasing decisions. However, an equilibrium is a pooling equilibrium if both firms propose the same testing decisions. Thus, consumer cannot identify the firm's types in making their purchasing decisions.

Definition 2 - A Perfect Bayesian Equilibrium $(\psi_g, \psi_b, \Omega, \beta(.))$ is separating if $\sigma_g = 1$ and $\sigma_b = 0$ and pooling if $\sigma_g = \sigma_b = 1$.

We are mainly interested in the separating equilibrium $(g, b) = (t, n)$, i.e., when green firm seeks certification and brown firm doesn't seek certification as the other separating equilibrium, $(g, b) = (n, t)$, is a wrong signal of quality. Also, the pooling equilibrium $(g, b) = (n, n)$ is not explicitly solved as our analysis assumes only 1 firm is certified. For (n, n) case to be a pooling equilibrium, we need to drop the assumption that at least one firm is certified, so it comes out to be status quo.

3.1 Separating Equilibrium

A separating equilibrium is where the firm offering a high quality product conveys a signal about its quality and that it cannot be mimicked by the low quality firm. Under separating equilibrium green firm seeks certification and brown firm doesn't seek certification, i.e., $\sigma_g = 1$ and $\sigma_b = 0$. Thus, certification helps consumers' to distinguish between the high and low quality product.

Definition 3 - The strategy of the green firm $\psi_g = \{\sigma_g, q_g\}$ and brown firm $\psi_b = \{\sigma_b, q_b\}$ is a separating equilibrium if and only if -

(a) $\sigma_g = 1$ and $\sigma_b = 0$.

(b) Each firm maximises profits to choose quantity, given others players strategy.

(c) $\pi_g(t, n) \geq \pi_g(n, n)$ and $\pi_b(t, n) \geq \pi_b(t, t)$

Condition (a) states that the green firm decides to seek certification test, i.e., $\sigma_g = 1$ and brown firm decides not to seek certification, i.e., $\sigma_b = 0$. Condition (b) implies firms compete in quantities. Condition (c) is an incentive compatibility condition that implies that green firm obtains a higher profit from seeking certification and the brown firm obtains a higher profit from not seeking certification. That is given brown firm chooses (n), the green firm profits are higher when it chooses (t) than when it chooses (n). Similarly, given green firm chooses (t), the brown firm profits are higher when it chooses (n) than when it chooses (t). The next proposition derives the conditions under which separating equilibrium will hold true.

Proposition 3: The separating equilibrium (t, n) of the game in which green firm ask for certification and brown firm doesn't ask for certification is a Perfect Bayesian Equilibrium if $Max(\underline{C}_{tn}, 0) \leq C \leq \overline{C}_{tn}$ holds, .i.e.,

$$Max(\underline{C}_{tn} = \frac{\phi_b((1-\phi_g)(2s_g - s_b)^2 s_g - \phi_g s_g^2 s_b)}{(4s_g - s_b)^2}, 0) \leq C \leq \frac{\phi_g(2s_g - s_b)^2 s_g - s_g^2 s_b}{(4s_g - s_b)^2} = \overline{C}_{tn}$$

Proof: See Appendix A.3

The lowest certification fee for the separating equilibrium is given by $Max(\underline{C}_{tn}, 0)$. When $\phi_g \geq \frac{4s_g^2 + s_b^2 - 4s_g s_b}{4s_g^2 + s_b^2 - 3s_g s_b}$, the lowest certification fee for the separating equilibrium to exist is 0. We observe that the certification fees would be zero only for sufficiently high values of ϕ_g or $\phi_b = 0$. For example, given $s_g = 100, s_b = 2$, when the probability of green firm passing the test is sufficiently high ($\phi_g \geq 0.9949$), then minimum certification fees is zero for separating equilibrium (t, n) to exist. Thus, even for certification fees as low as 0, the brown firm doesn't prefer to seek certification as its profits are higher when there is differentiation.

3.2 Pooling Equilibrium

A pooling equilibrium is where both firms are proposing same testing decisions. The high quality firm is unable to convey a signal about its quality and the consumers cannot distinguish between the firms products in making their purchase decisions. Note that "pooling equilibrium" is in the sense that both firms seek or do not seek certification and not on whether they receive or not receive certification. It may still be possible that consumers see a certified and a non-certified product.[3] Under pooling equilibrium, both green firm and brown firm seek certification, i.e., $\sigma_g = 1$ and $\sigma_b = 1$.

Definition 4 - The strategy of the Green Firm $\psi_g = \{\sigma_g, q_g\}$ and brown firm $\psi_b = \{\sigma_b, q_b\}$ is pooling equilibrium if and only if -

(a) $\sigma_g = 1$ and $\sigma_b = 1$.

(b) Each firm maximises profits to choose quantity, given others players strategy.

[3] To be more precise, we can call it Partial Pooling Equilibrium, but here we call it Pooling Equilibrium.

(c) $\pi_g(t,t) \geq \pi_g(n,t)$ and $\pi_b(t,t) \geq \pi_b(t,n)$

Condition (a) states that the green and brown firm both seek certification test, i.e., $\sigma_g = 1$ and $\sigma_b = 1$. Condition (b) implies each firm simultaneously choose quantity. Condition (c) implies that given brown firm chooses (t), the green firm profits are higher when it chooses (t) than when it chooses (n). Similarly, given green firm chooses (t), the brown firm profits are higher when it chooses (t) than when it chooses (n). Thus, in the next proposition calculates the conditions under which pooling equilibrium will hold true.

Proposition 4: The pooling equilibrium (t,t) of the game in which green and brown firm ask for certification is a Perfect Bayesian Equilibrium if $0 \leq C \leq \overline{C}_{tt}$ holds,.i.e.,

$$0 \leq C \leq \overline{C}_{tt} = \frac{\phi_b((1-\phi_g)(2E(s|c)-E(s|un))^2 E(s|c) - \phi_g(E(s|c))^2 E(s|un))}{(4E(s|c)-E(s|un))^2}$$

where $E(s|c) = \frac{\phi_g(1-\phi_b)s_g+\phi_b(1-\phi_g)s_b}{\phi_g(1-\phi_b)+\phi_b(1-\phi_g)}$ and $E(s|un) = \frac{\phi_b(1-\phi_g)s_g+\phi_g(1-\phi_b)s_b}{\phi_g(1-\phi_b)+\phi_b(1-\phi_g)}$.

Proof: See Appendix A.4

The above proposition shows for both firms to seek the certification fees should be sufficiently small.

Thus, when there is separating equilibrium, consumers are able to identify the green and brown firm product in their purchasing decisions. However, under pooling equilibrium, when both the firms seek certification, the consumers are unable to distinguish between the firms product. The situation in which the probability of green firm passing the test is very high and brown firm passing the test is low; the situation resembles that of perfect certification. In this paper, we have shown that for the case of perfect certification, separating equilibrium may exists when the certification fees is close to zero. In such a situation, subsidizing certification fees would encourage firms to seek certification and information would be disclosed. However, when there is imperfect certification, where probabilities of the firms passing the test are in the interior ranges, then certification fees for separating equilibrium to exist is high. Thus, subsidizing certification wouldn't be a recommended policy.

A numerical example to illustrate this result is the following - when $\phi_g = 0.5$, $\phi_b = 0.2$, $s_g = 100$, $s_b = 20$, the minimum certification fees for separating equilibrium to exists is 2.11 and for pooling equilibrium to exists the certification fees should be less than 1.34. Subsidizing certification might move separating equilibrium to pooling equilibrium and thus, it is not optimal to subsidize certification.

Our results are in line with Mason (2011), and Bottega and Freitas (2013), which showed under imperfect certification separating equilibrium exists if the certification costs is moderately large or the probability of brown firm passing the test is sufficiently small. The pooling equilibrium exists if the certification costs are sufficiently small.

3.3 Graphical Analysis

We do numerical analysis to compare the range of certification fees for separating and pooling equilibrium. The ranges are not comparable analytical, thus, we discuss the results below through numerical simulation.

Figure 3 shows for given $s_g = 100, s_b = 20, \phi_b = 0.2$, how the optimal fees changes under separating and pooling equilibrium for different values of ϕ_g. For given values of s_g, s_b and ϕ_b, as ϕ_g increases, i.e., the probability of green firm passing the test increases, the range of separating equilibrium, i.e., $\underline{C}_{tn}$ and $\overline{C}_{tn}$ increases. With ϕ_g close to 0.9, the minimum certification fees for separating equilibrium to exists, i.e., ($\underline{C}_{tn}$) is close to 0. This is because as the probability of green firm passing the test increases, brown firm has the incentive to differentiate the product by not labelling. For given values of s_g, s_b and ϕ_b, the certification fees needs to be sufficiently low for pooling equilibrium to exist.

Figure 4 shows for given $s_g = 100, s_b = 20, \phi_g = 0.8$, how the optimal fees changes for different values of ϕ_b. For given values of s_g, s_b and ϕ_g , as ϕ_b increases, i.e., the probability of brown firm passing the test increases, the minimum certification fees ($\underline{C}_{tn}$) required for separating equilibrium to exist increases. Also, the maximum certification fees ($\overline{C}_{tn}$) for separating equilibrium to exist doesn't depend on ϕ_b. For given values of s_g, s_b and ϕ_g, the certification fees needs to be sufficiently low for pooling equilibrium to exist. However, this value is comparatively higher for large values of s_g.

4 CONCLUSION

Certification by third party acts as a information disclosure mechanism by providing informatiom on the hidden attributes of the product. Using vertical product differentiated model, the paper studied the role of certification when consumers are unable to observe the quality of the product. The paper first showed in the absence of labelling by the third party, there is problem of adverse selection, i.e., only brown firm produce in the market. The above result holds for sufficiently large values of k and large gap between the two qualities. Thus, due to credence nature of the products, market fails to provide pareto optimum.

Third party certification is a possible solution for the market failure resulting from information asymmetry problem. As it costly to monitor continuously, there is random monitoring by the certification agency. Next,under imperfect certification, the paper found the range of the certification fees for separating and pooling equilibrium to exist with quantity competition. Under separating equilibrium, the two firms choose different testing decisions, i.e., green firm seek certification and brown firm doesn't seek certification. The consumers are able to differentiate between the firms product. It is found that when the quality differential increases and probability of green firm passing the test is sufficiently high, the minimum certification fees for the separating equilibrium to exist is zero. Even for certification fees close to zero, brown firm doesn't prefer to seek certification as its profits are higher with differentiation. Thus, in the Perfect Bayesian Equilibrium of the game, label allows firms to benefit from differentiation. However, under pooling equilibrium, both firms choose same testing decisions, i.e., both green and brown firm seek certification. For both firms to seek the certification, fees should be sufficiently small. The paper shows that under imperfect certification it is not optimal to subsidize certification. This is in contrast with perfect certification, where the policy of subsidization helps.

It is worth noting that we have conducted my research assuming duopoly market structure and quality as exogenous variable. For future research, it would be an interesting area to extent the analysis to “n” firms under quantity competition. Also, the analysis could be extended by treating quality as an endogenous variable.

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APPENDIX

A Proof of Propositions

A.1 Proof of Proposition 1

(a) As $s_g > s_b$, it follows trivally from equation (3) and (4) that $q_g^* > 0$ and $q_b^* > 0$.

(b) As $s_g > s_b$, we have $2s_g^2 - s_b s_g > s_b s_g$. Thus, it follows $p_g^* > p_b^*$ from equation (5) and (6). Also, as $s_g > s_b$, we have $2s_g - s_b > s_g$. Thus, it follows $q_g^* > q_b^*$ from equation (3) and (4).

(c) The revenue of the green firm is increasing in s_g, given s_b and is decreasing in s_b, given s_g, i.e.,

$$\begin{aligned} \frac{\partial R_g}{\partial s_g} &= \frac{(2s_g - s_b)(2s_g(4s_g - s_b) + s_b^2)}{(4s_g - s_b)^3} > 0 \\ \frac{\partial R_g}{\partial s_b} &= \frac{-4s_g^2(2s_g - s_b)}{(4s_g - s_b)^3} < 0 \end{aligned}$$

The revenue of the brown firm is increasing in s_b, given s_g and is decreasing in s_g, given s_b, i.e.,

$$\begin{aligned} \frac{\partial R_b}{\partial s_b} &= \frac{s_g^2(4s_g + s_b)}{(4s_g - s_b)^3} > 0 \\ \frac{\partial R_b}{\partial s_g} &= \frac{-2s_g s_b^2}{(4s_g - s_b)^3} < 0 \end{aligned}$$

A.2 Proof of Proposition 2

Using equation (10) and the equilibrium quantity of green and brown firm (equation (14), (15)), price of a good is given as -

$$p = \frac{s_g + s_b + A}{9} \tag{29}$$

On the substituting the values of q_g (equation (14)), and p (equation (29)) in the profit equation of green firm (i.e., equation (11)) , we get -

$$\begin{aligned}\pi_g &= \frac{2}{3} + \frac{s_g + s_b + A}{9}(\frac{2s_b - 4s_g + 2A}{9(s_g - s_b)}) - ks_g \\ &= \frac{2(s_g + s_b + A)}{27} - B - ks_g \end{aligned} \tag{30}$$

On the substituting the values of q_b (equation (15)), and p (equation (29)) in the profit equation of brown firm (i.e., equation (11)) , we get -

$$\begin{aligned}\pi_b &= \frac{(s_g + s_b + A)(4s_g - 2s_b - 2A)}{81(s_g - s_b)} - ks_b \\ &= \frac{2(s_g + s_b + A)(2s_g - s_b - A)}{81(s_g - s_b)} - ks_b = B - ks_b \end{aligned} \tag{31}$$

Thus, the result of the proposition follows from equations (30) and (31). For case (a), when both firms produce in the market, we have $\pi_g \geq 0$ and $\pi_b \geq 0$. For case (b), when only brown firm produces in the market ,i.e., there is adverse selection, we have $\pi_g < 0$ and $\pi_b \geq 0$. For case (c), none of the firms produce in the market, i.e., $\pi_g < 0$ and $\pi_b < 0$.

A.3 Proof of Proposition 3

Substituting $\sigma_g = 1$ and $\sigma_b = 0$ in equation (19), (20), (21) and (22), we get $\beta(s_g|c) = 1$, $\beta(s_g|un) = 0$, $\beta(s_b|c) = 0$ and $\beta(s_b|un) = 1$. Substituting these probabilities in equation (17) and (18), we get $E(s|c) = s_g$ and $E(s|un) = s_b$.

Condition 1 - $\pi_g(t, n) \geq \pi_g(n, n)$

Substituting the values of the profits calculated, we get -

$$\frac{\phi_g(2E(s|c) - E(s|un))^2 E(s|c)}{(4E(s|c) - E(s|un))^2} - ks_g - C \geq \frac{(E(s|c))^2 E(s|un)}{(4E(s|c) - E(s|un))^2} - ks_g$$

$$\frac{\phi_g(2E(s|c)-E(s|un))^2E(s|c)-(E(s|c))^2E(s|un)}{(4E(s|c)-E(s|un))^2}\geq C$$

Substituting $E(s|c)=s_g$ and $E(s|un)=s_b$, the condition 1 is -

$$\frac{\phi_g(2s_g-s_b)^2s_g-s_g^2s_b}{(4s_g-s_b)^2}\geq C \tag{32}$$

<u>Condition 2</u> - $\pi_b(t,n)\geq\pi_b(t,t)$

Substituting the values of the profits calculated, we get -

$$\frac{\phi_g(E(s|c))^2E(s|un)}{(4E(s|c)-E(s|un))^2}-ks_b\geq\frac{\phi_b(1-\phi_g)(2E(s|c)-E(s|un))^2E(s|c)+\phi_g(1-\phi_b)(E(s|c))^2E(s|un)}{(4E(s|c)-E(s|un))^2}-ks_b-C$$

$$C\geq\frac{\phi_b(1-\phi_g)(2E(s|c)-E(s|un))^2E(s|c)-\phi_g\phi_b(E(s|c))^2E(s|un)}{(4E(s|c)-E(s|un))^2}$$

Substituting $E(s|c)=s_g$ and $E(s|un)=s_b$, condition 2 is -

$$C\geq\frac{\phi_b(1-\phi_g)(2s_g-s_b)^2s_g-\phi_g\phi_bs_g^2s_b}{(4s_g-s_b)^2} \tag{33}$$

Thus, from equation (32) and (33), result is derived. The separating equilibrium (t,n) exists when C lies between $(Max(\underline{C}_{tn},0),\overline{C}_{tn})$.

A.4 Proof of Proposition 4

Substituting $\sigma_g=1$ and $\sigma_b=1$ in equation (19), (20), (21) and (22), we get $\beta(s_g|c)=\beta(s_b|un)=\frac{\phi_g(1-\phi_b)}{\phi_g(1-\phi_b)+\phi_b(1-\phi_g)}$ and $\beta(s_g|un)=\beta(s_b|c)=\frac{\phi_b(1-\phi_g)}{\phi_g(1-\phi_b)+\phi_b(1-\phi_g)}$. Substituting these probabilities in equation (17) and (18), we get the values of $E(s|c)$ and $E(s|un)$ defined in proposition 4.

<u>Condition 1</u> - $\pi_g(t,t)\geq\pi_g(n,t)$

Substituting the values of the profits calculated, we get -

$$\frac{\phi_b(1-\phi_g)(E(s|c))^2E(s|un)+\phi_g(1-\phi_b)(2E(s|c)-E(s|un))^2E(s|c)}{(4E(s|c)-E(s|un))^2}-ks_g-C \geq \frac{\phi_b(E(s|c))^2E(s|un)}{(4E(s|c)-E(s|un))^2}-ks_g$$

$$C \leq \frac{\phi_g(1-\phi_b)(2E(s|c)-E(s|un))^2E(s|c)-\phi_b\phi_g(E(s|c))^2E(s|un)}{(4E(s|c)-E(s|un))^2} \tag{34}$$

<u>Condition 2</u> - $\pi_b(t,t) \geq \pi_b(t,n)$

Substituting the values of the profits calculated, we get -

$$\frac{\phi_b(1-\phi_g)(2E(s|c)-E(s|un))^2E(s|c)+\phi_g(1-\phi_b)(E(s|c))^2E(s|un)}{(4E(s|c)-E(s|un))^2}-ks_b-C \geq \frac{\phi_g(E(s|c))^2E(s|un)}{(4E(s|c)-E(s|un))^2}-ks_b-C$$

$$C \leq \frac{\phi_b(1-\phi_g)(2E(s|c)-E(s|un))^2E(s|c)-\phi_g\phi_b(E(s|c))^2E(s|un)}{(4E(s|c)-E(s|un))^2} = \overline{C}_{tt} \tag{35}$$

Thus, the pooling equilibrium exists when $0 \leq C \leq \overline{C}_{tt}$, given by equation (35).

B Analysis of solution under Absence of Eco-Labelling

Simulatenously solving the best response functions of green and browm firm on Matlab, we get the following solutions -

(a) $q_g = 0, \quad q_b = 0$

This cannot be the solution, as it is not defined for the best response functions of green and brown firm.

(b) $q_g = \frac{-s_b}{s_g - s_b}, \quad q_b = \frac{s_g}{s_g - s_b}$

As $s_g > s_b$, we have $q_g < 0$. Thus, this is not a feasible solution.

(c) $q_g = \frac{2}{3} - \frac{8s_g - 4s_b + 4(s_g^2 + s_b^2 - s_g s_b)^{\frac{1}{2}}}{18(s_g - s_b)}, \quad q_b = \frac{8s_g - 4s_b + 4(s_g^2 + s_b^2 - s_g s_b)^{\frac{1}{2}}}{18(s_g - s_b)}$

Here, $q_g < 0$ and $q_b > 0$ for all values of s_g and s_b. Thus, this is not a feasible solution.

(d) $q_g = \frac{4s_b - 8s_g + 4(s_g^2 + s_b^2 - s_g s_b)^{\frac{1}{2}}}{18(s_g - s_b)} + \frac{2}{3}, \quad q_b = \frac{-(4s_b - 8s_g + 4(s_g^2 + s_b^2 - s_g s_b)^{\frac{1}{2}})}{18(s_g - s_b)}$

Here, $q_g > 0$ and $q_b > 0$ for all values of s_g and s_b. Thus, the unique Nash Equilibrium is given by solution (d).

C Second Order Conditions for Profit Maximisation under Absence of Eco-Labelling

The second order condition for Green and Brown firm is given below -

$$\frac{\partial^2 \pi_g}{\partial q_g^2} = \frac{-2q_g^3 s_g - 6q_g^2 q_b s_g - 6q_g q_b^2 s_g + 2q_b^2 s_g - 2q_b^2 s_b - 2q_b^3 s_g}{(q_g + q_b)^3} < 0$$

$$\frac{\partial^2 \pi_b}{\partial q_b^2} = \frac{-2q_b^3 s_b - 6q_b^2 q_g s_b - 6q_b q_g^2 s_b + 2q_g^2 s_b - 2q_g^2 s_g - 2q_g^3 s_b}{(q_g + q_b)^3} < 0$$

For both the firms, the second order condition for profit maximisation is satisfied, as the numerator is always negative, and the denominator is always positive.

D Second Order Conditions for Profit Maximisation under Third Party Certification

The second order condition for profit maximisation is satisfied for both Certified and uncertified firm,given below -

$$\frac{\partial^2 R_c}{\partial q_c^2} = -2E(s|c) < 0$$

$$\frac{\partial^2 R_{un}}{\partial q_{un}^2} = -2E(s|un) < 0$$

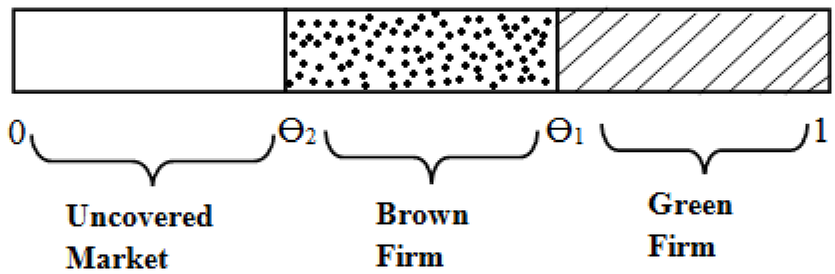

Figure 1: **Covered and Uncovered Market**

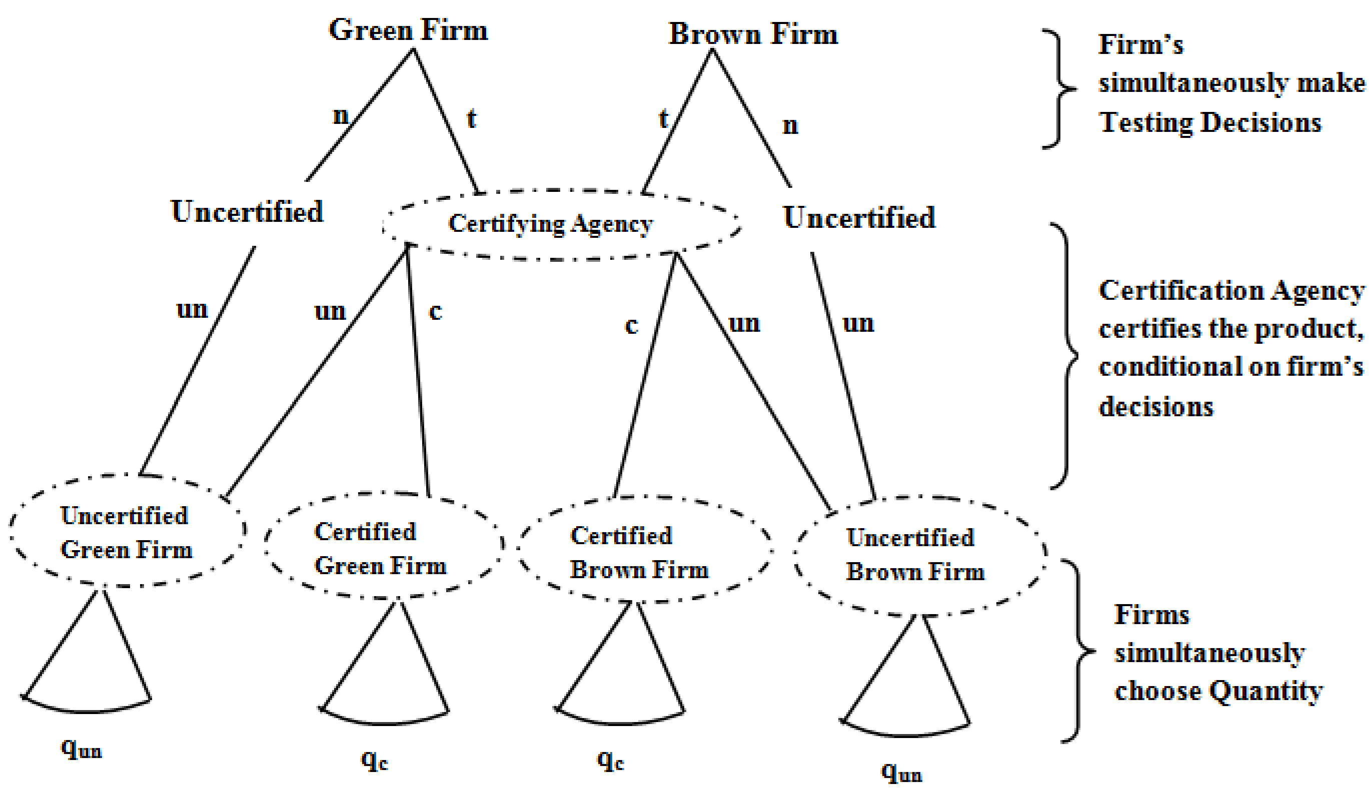

Figure 2: **Extensive Form Game under Third Party Certification**

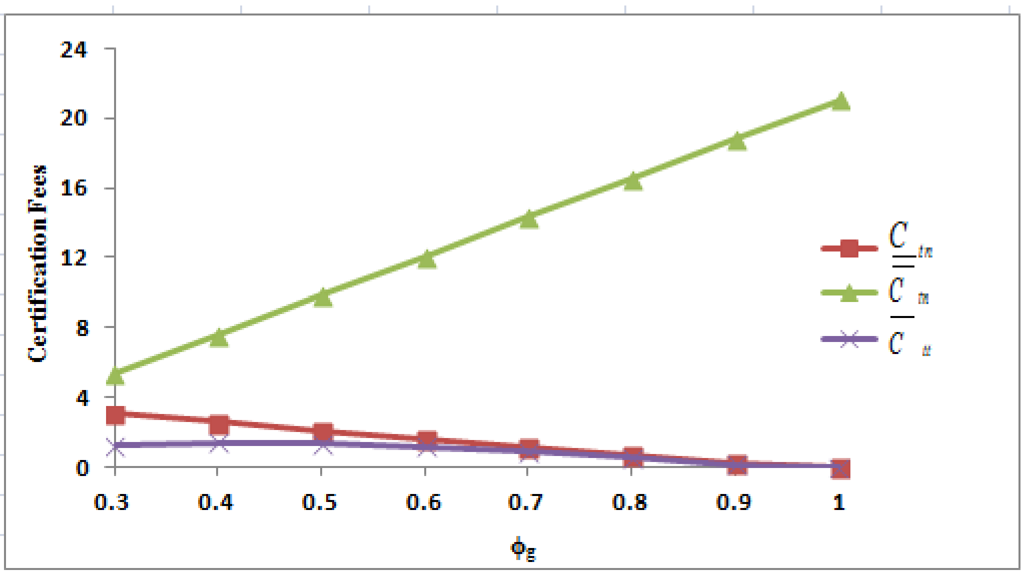

Figure 3: **Certification Fees for Separating Equilibrium** $(Max(\underline{C}_{tn}, 0) < C < \overline{C}_{tn})$ **and Pooling Equilibrium** $(0 < C < \overline{C}_{tt})$**, given** $s_g = 100, s_b = 20, \phi_b = 0.2$

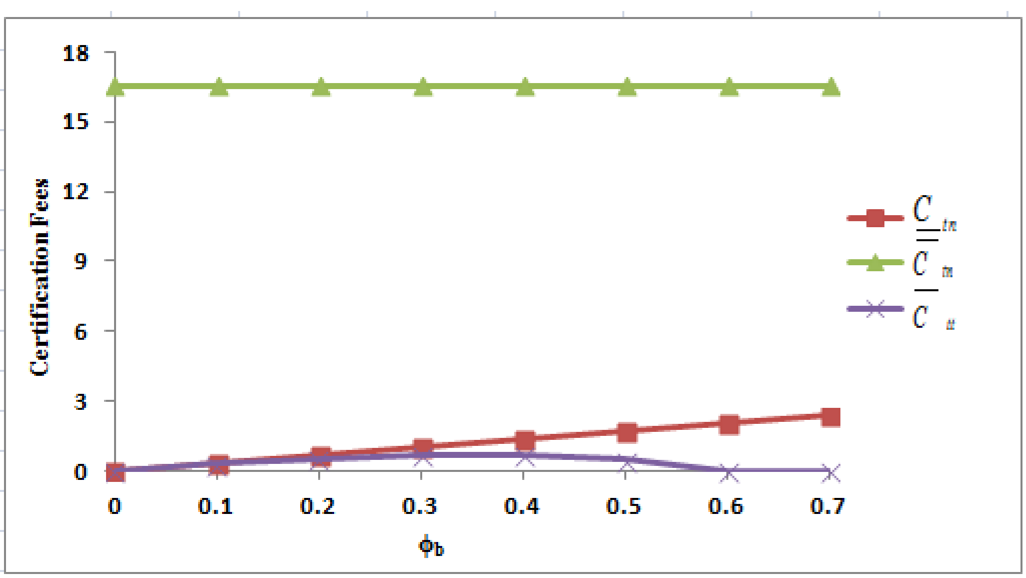

Figure 4: **Certification Fees for Separating Equilibrium** $(Max(\underline{C}_{tn}, 0) < C < \overline{C}_{tn})$ **and Pooling Equilibrium** $(0 < C < \overline{C}_{tt})$**, given** $s_g = 100, s_b = 20, \phi_g = 0.8$